de Sousa, R. T., dos Santos, M. G. M., & Alves, F. R. V. (2023). The theory of didactic situations and the generalization of pythagoras' theorem: An experience mediated by geogebra software. *Journal of Research in Science and Mathematics Education (J-RSME), 2*(3), 147-159.

Original scientific paper https://doi.org/10.56855/jrsme.v2i3.731

Received: 04 August 2023.
Revised: 18 October 2023.
Accepted: 22 November 2023.

The Theory of Didactic Situations and the Generalization of Pythagoras' Theorem: An Experience Mediated by GeoGebra software

Renata Teófilo de Sousa[1*], Maria Graciene Moreira dos Santos[2], Francisco Régis Vieira Alves[3]

[1] Ceará Education Department (SEDUC), Fortaleza, Brazil

[2,3] Federal Institute of Education, Science and Technology of Ceará, Fortaleza, Brazil

Abstract

Purpose: The objective of this work is to present the development of an activity on the Generalization of Pythagoras' Theorem with the support of GeoGebra. **Methodology:** The methodology used was Didactic Engineering, in which we conducted a qualitative and exploratory analysis of the data. For the teaching session, we relied on the Theory of Didactic Situations. The activity was developed with 18 students from the 2nd year of high school at a Professional School in Sobral, Ceará, Brazil, in April 2023. The students were selected because they faced difficulties with the topic, identified through a school diagnostic assessment. Its implementation was an intervention by the teacher to understand these difficulties and clarify them with the support of GeoGebra. **Findings:** In the results, specific difficulties of the students were identified, and it was observed that the Generalization of Pythagoras' Theorem via GeoGebra software was well-received by the group. **Significance:** It awakened their perception of the elements that make up a right triangle.

Keywords: Didactic Engineering, GeoGebra, Pythagoras' Theorem, Theory of Didactic Situations.

* Corresponding author: Renata Teófilo de Sousa, *rtsnaty@gmail.com*

Introduction

Digital Technologies as a methodological resource in the teaching and learning process is a reality at all levels of education, as "[...] society is moving towards being a society that learns in new ways, through new paths, with new participants (actors), continuously" (Moran, 2007, p.11). Thus, the National Common Curricular Base (BNCC, acronym in Portuguese) highlights its importance and how necessary its inclusion is for the success of learning, bringing the school closer to reality and the changes caused by the advance of technology in contemporary society (Brazil, 2018).

With regard to learning difficulties, Mathematics is still a feared subject with low performance. Geometry is considered one of its curricular components which, even with its proved relevance, is still an obstacle for students (Sousa et al, 2021). Some difficulties are recurrent, such as the geometric perception of a shape linked to the mathematical concepts involved in a problem.

Thus, in this work we present the contribution of the GeoGebra software to geometric understanding, since its use allows the student to visualize non-trivial elements when interpreting a problem, in addition to its usability, being a free software and easy to manipulate. GeoGebra provides support for teaching methodology and assists student understanding and learning through visual and manipulable resources (Abar, 2020). Furthermore, its use as a resource allows the visualization of imaginable situations, which the student would not be able to see in a problem done by hand, with pencil and paper (Alves & Borges Neto, 2012; Sousa et al, 2021).

The objective of this work was to present the development of a didactic sequence for teaching the Generalization of the Pythagoras' Theorem with support of GeoGebra, exploring the student's geometric perception. The choice of this topic was based on the absence of this generalization in the most part of Brazilian textbooks for high school.

As a methodology we used Didactic Engineering (DE) (Artigue, 1996), with data analysis based on internal validation and presented in the form of an experience report. To structure the teaching session, we adopted the Theory of Didactic Situations (TDS), because this theory allows the student to be a protagonist in the construction of their own knowledge (Brousseau, 2008). Therefore, in the following sections, we present the development of the four phases of DE, guided by the assumptions of TDS, followed by the authors' notes and considerations.

Method

The methodology adopted in this work was Didactic Engineering (DE) (Artigue, 1996), with the aim of studying and understanding the teaching and learning process of the Generalization of the Pythagoras' Theorem, aiming at the design and development of a didactic sequence for its teaching. The DE is structured into four phases, which are, namely: (i) preliminary analysis; (ii) design and *a priori* analysis; (iii) experimentation, and (iv) *a posteriori* analysis and validation. In the case of this research, we performed an internal validation, based on the analysis of what happens in the classroom.

In the preliminary analysis of this work, we sought to understand how the Pythagoras' Theorem approach occurs in textbooks and classes. Based on this, in *a priori* analysis phase we present a design with a construction on the generalization of Pythagoras' Theorem in GeoGebra software, guided by the assumptions of TDS. In the experimentation phase, we collected data and observed the students' behavior and resourcefulness in the activity, while in *a posteriori* analysis and validation, we compared what was planned *a priori* and what was executed *a posteriori*, seeking an internal validation of the data.

The development of the teaching session took place at a State School of Professional Education (EEEP) in the state of Ceará, Brazil. The target audience was a group of 18 students in the 2nd year of high school, who had difficulties to understand the Pythagoras' Theorem in problems. These difficulties were identified in an internal diagnostic assessment, which is carried out by the school every two months. The activity was developed in two classes of 50 minutes each, aimed at curricular reorganization[1], linked to the Study Schedule (SS) discipline, at the school's laboratory, in the first semester of 2023.

We highlight that this study was developed based on the implementation of a didactic proposal constructed by Santos and Alves (2022), in which we referenced and sought to verify the hypotheses suggested in the study in a practical way in the classroom. Each of the following sections provides notes on the four phases of Didactic Engineering developed, from the brief presentation of the theoretical framework to the conception of the didactic situation, data collection and comparison between *a priori* and *a posteriori* analysis.

Preliminary analysis

In this stage we bring a theoretical survey on how knowledge is taught to the student, their ideas and difficulties about the object taught, in addition to the obstacles in the context of their teaching, based on a bibliographical review on the topic (Artigue, 1996). Here we highlight the specific difficulties in relation to the Pythagorean Theorem and its generalization and the BNCC proposal for teaching with technologies.

One of the main difficulties students face in Geometry is associating geometric concepts with real practice, which also occurs in our specific mathematical object. In the research by Pereira et al. (2016), regarding the main errors made by students in relation to the Pythagoras' Theorem, the authors highlight three, namely:

(a) Difficulty in correctly identify the elements that make up a right-angled triangle (cathetus and hypotenuse).
(b) Error in the interpretation and application of rules and strategies.
(c) Errors in the development of basic mathematical operations.

In addition, Cruz (2015, p. 17) detected other difficulties in his research, such as "the use of the theorem to calculate the third side of a non-rectangle triangle; understand the statements of mathematics problems and prepare an answer with arguments articulated within a coherent text". In both studies, the absence of an approach to this topic in most textbooks, especially with the use of technology, is highlighted. It is mentioned that the Pythagoras' Theorem is addressed, but without explaining its generalization, with models, questions, and problems at different levels, and also without a technological approach (Cruz, 2015; Pereira et al., 2016).

Such difficulties are recurrent in Basic Education and, aiming to minimize them, BNCC suggests approaches via software (Brazil, 2018), to explore the most varied themes from the perspective of Dynamic Geometry, such as GeoGebra, highlighted in this work.

Oliveira and Leivas (2017) state that Geometry, due to its visual nature, has the potential to develop the student's perception and autonomy of reasoning, being able to detach itself from ready-made

[1] The curricular reorganization in state schools in Ceará refers to the approach to curricular content from previous years, but which were not properly learned due to the setbacks imposed by the COVID-19 pandemic period.

structures and formulas. In parallel, the relationship between Geometry and Technology, recommended by the BNCC, highlights the relevance of this association for student development, through investigative activities that interrelate geometric knowledge and reality, via problem solving (Brazil, 2018).

Based on the notes about the references mentioned, in the following section we present the elaboration of an a priori didactic situation, guided by the assumptions of TDS.

***A priori* analysis**

In this section we present the teaching proposal for demonstrating the generalization of the Pythagorean Theorem, based on the construction of a didactic situation with the support of the GeoGebra software and structured based on the Theory of Didactic Situations.

The Theory of Didactic Situations (TDS) aims to bring the student's work closer to a researcher, through the formulation of hypothesis. Brousseau (2002) explains that teaching situations must be strategically produced by the teacher, aiming at a *milieu* (environment) for the student to construct and appropriate knowledge by himself. The learning process in TDS is divided into dialectics, which are situations of action, formulation, validation, and institutionalization. The first three are called *adidactic phase*: they are the moment in which the student interacts with the problem and produces knowledge, without intervention directly from the teacher (Almouloud, 2007; Brousseau, 2008).

For a didactic situation to be well developed, it must be preserved in what Brousseau (2008) brings as a didactic contract. The author explains that the didactic contract functions as a verbal agreement, with this relationship being mediated by knowledge, which aims to determine the roles of the subjects whose reciprocity is necessary. For the teaching session, we present a didactic sequence and then suggest its development in GeoGebra. At https://www.geogebra.org/m/pr8jpnkc we find the construction developed and in Table 1 we present the proposed activity:

Table 1

Proposed Didactic Sequence

Consider a right-angled triangle, with polygons formed on each of its sides:
a) Calculate the areas of polygons using the measurements of the sides of the triangles. Consider that a = AC, b = BC and c = BC. Set the value of each polygon.
b) Add the result of the two smaller polygons.
c) Compare the result with the value of the larger polygon. What happened?
d) Move the slider *n* and perform the same steps on all polygons shown.
e) Based on the tests carried out, was the relationship $a^2 = b^2 + c^2$ (Pythagorean Theorem) true? Justify.

This activity aims to establish a relationship between the areas of polygons built on the sides of a right-angled triangle, achieving the geometric perception of the Generalization of Pythagoras' Theorem, encouraging the use of GeoGebra as a facilitator of the teaching-learning process. In the exemplified construction (Figure 1), we highlight the area of polygons in different colors, as a way of exploring the visual prism:

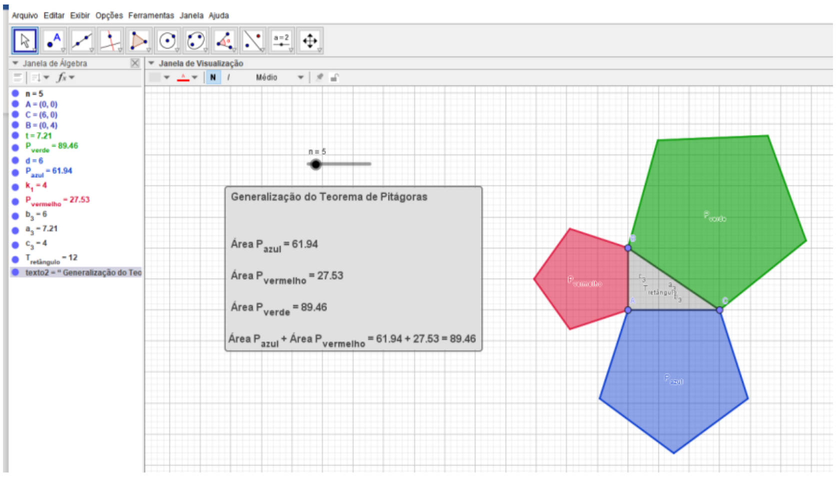

Figure 1. Proposed construction with GeoGebra.

The resolution of the didactic sequence must follow the TDS route, in which the teacher can choose to present the construction made for the student to manipulate, or build it with the class, depending on their class planning, time and resources.

In the *action situation*, we hope that students, when interacting with the proposed situation, observe the mathematical elements and based on their previous knowledge about areas of flat figures, prepare to carry out the construction steps in GeoGebra, mediated by the teacher. It is a simple sequence, with trivial tools, not requiring prior and/or advanced knowledge of the software.

In the *formulation situation* and after construction in GeoGebra, students are expected to manipulate the slider, observing what happens with the construction and seeking to execute what is asked in the didactic sequence. When calculating the area of polygons, the student can use the software's Area tool, or perform manual procedures. By manipulating the slider, they may come across the main regular polygons, as illustrated in Figure 2:

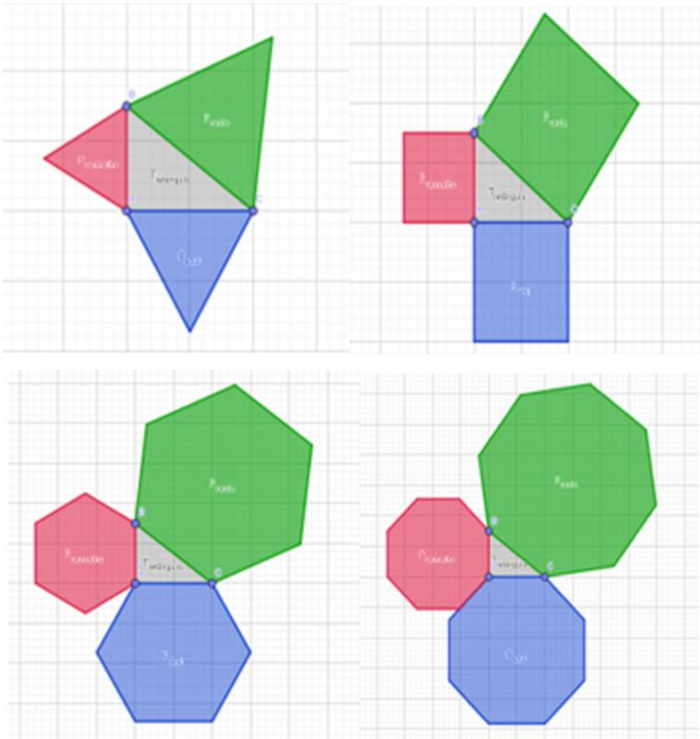

Figure 2. Examples of polygons viewed by the student using slider n.

By calculating the area of the polygons and establishing some conjectures, we hope that in the *validation situation* the students will conclude that Pythagoras' Theorem is valid and that the sum of the areas of the polygons built on the cathetus is equal to the area of the polygon built on the hypotenuse.

Geometry classes still give priority to flat space, using the most well-known flat figures and polygons, and other types of shapes are present in our daily lives. Regarding Spatial Geometry, in particular, students have extensive difficulties, primarily in relation to visualization and representation, as they recognize few concepts of basic geometry and, therefore, spatial geometry. They also present problems in perceiving the relationships between objects and identifying the properties of the figures that form solids, among other concepts.

With this in mind, in Figure 3, as a curiosity, we also illustrate the same constructions in the 3D window:

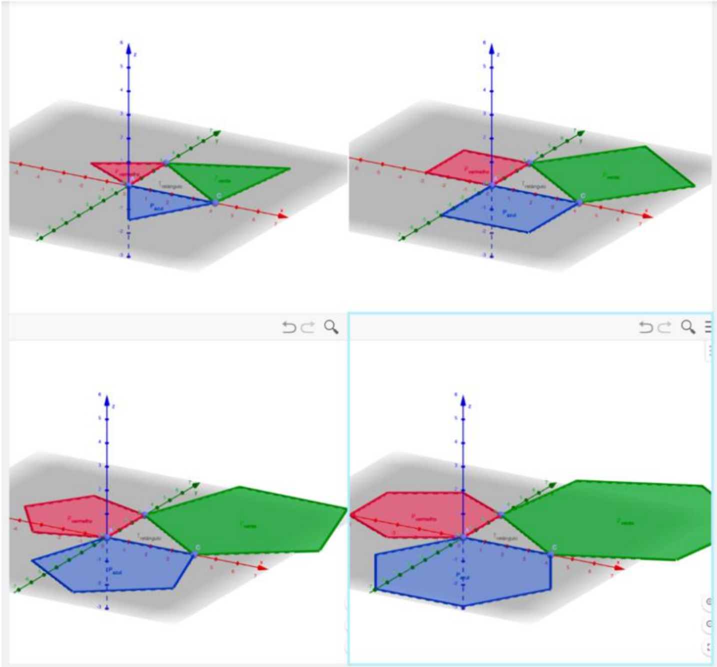

Figure 3. The polygons viewed by the student at 3D window.

Next, it is expected that in the *situation of institutionalization*, the teacher validates the model proposed by the students, checking whether the calculation of areas was done manually or via software, correcting inadequate models and formalizing the mathematical concept regarding the generalization of the Pythagoras' Theorem.

Experimentation

In this step, we present a description of the teaching session. Given the brevity of this work, we present our experience very succinctly.

Initially, the teacher welcomed the class and established the didactic contract, highlighting the importance of everyone's participation, as well as trying to resolve the proposed activity. The teacher briefly presented GeoGebra and its guides, starting the step-by-step construction with the class, as follows in Table 2:

Table 2

Sequence of construction steps mediated by the teacher.

Consider a right-angled triangle, with polygons formed on each of its sides:
1. First, students constructed a right-angled triangle with sides 3, 4, and 5 using the polygon tool and entering the value of each side in the input field. 2. Then, they created the n slider, with minimum and maximum values equal to 3 and 20, respectively, and increment equal to 1. 3. After that, using the regular polygon tool, on each side of the right triangle, they constructed a polygon with all sides congruent and with a length equal to n, referring to what was marked in the slider.

In the *action situation*, when appropriating the activity statement and observing the construction, the students searched for the requested mathematical elements. Some prior knowledge was mobilized, such as plane figures and polygon areas, so that they could follow what the situation required.

In front of the construction in GeoGebra, the students interacted with the *milieu*. At first, their greatest curiosity was changing the slider, noticing (with admiration) the diversity of polygons built on the sides of the classic Pythagorean triangle.

From this stage onwards, the *formulation situation* began, where the students raised some questions and exchanged information with their colleagues and the milieu. The mediating teacher, in order to instigate the student's relationship with the environment, asked if they remembered how to calculate the area of those polygons, noting that some of the students only knew how to calculate the area of the triangle and the square. We have some records of the formulation situation in Figures 4, 5 and 6:

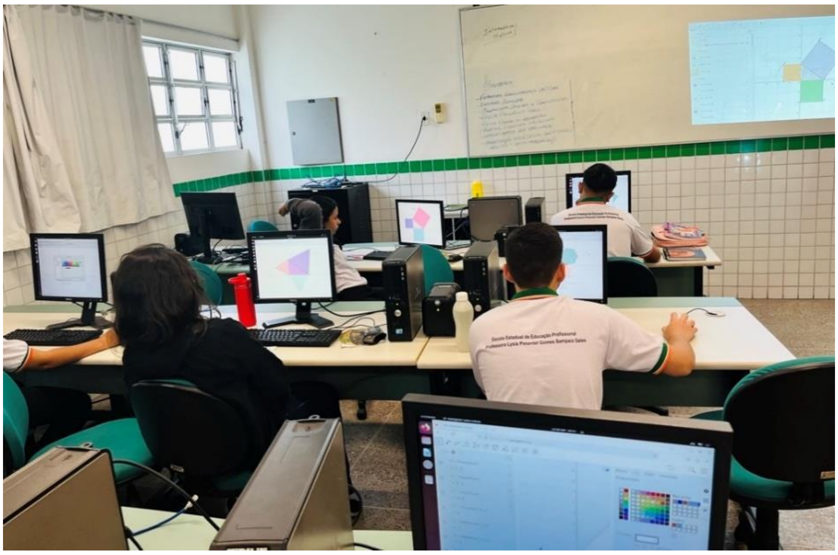

Figure 4. Formulation situation – photo record no. 1.

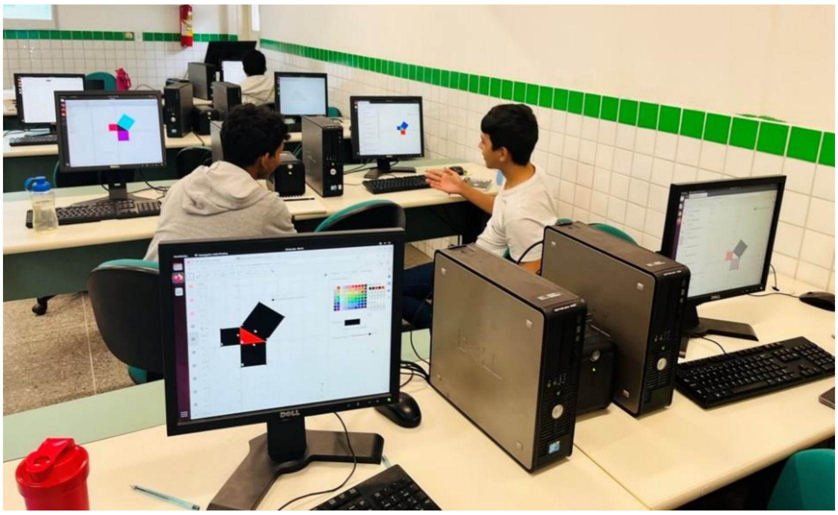

Figure 5. Formulation situation – photo record no. 2.

Figure 6. Intermediation of the teacher for the construction in GeoGebra.

As the students had difficulties in carrying out the task, the teacher encouraged them to use slider n. Based on this stimulus, the students were able to describe their strategies, some verbally, others written, but all with the support of GeoGebra in the construction of their conjectures. It is important to highlight that the teacher did not intervene to suggest the answer to the student, but rather encouraged him to build knowledge, as proposed in TDS (Brousseau, 2008).

In the *validation situation*, as predicted in *a priori* analysis, based on the propositions made in the didactic sequence presented, as well as interactions between peers, the students pointed out that the area of the larger polygon, built on the hypotenuse, will always be equivalent to the sum of the two areas of the smaller polygons, built on the cathetus.

Sousa et al. (2021, p. 117) highlight "the importance of students' visualization and perception with GeoGebra, as this resource allows the inference of information beyond what the question presents, becoming an element that facilitates geometric thinking". So, in this sense, it was really expected that from handling the slider and perceiving the different flat figures, as well as the relationship between their areas, that students would deduce the generalization of the Pythagoras' Theorem.

Thus, in the *institutionalization situation*, the mediating teacher entered the scene, to establish learning in a formal way, with appropriate mathematical language (Almouloud, 2007). The teacher used a mathematical model extracted from the work of Machado (2012, p. 139), which presents the Generalization of Pythagoras' Theorem as (Table 3):

Table 3

Mathematical concept used for institutionalization situation.

"If the square of the measurement of one of the sides of a triangle is equal to the sum of the squares of the measurements of the two other sides, then the triangle is right-angled, with the right angle opposite to the first side". We can also express this relationship as follows: *the area of the square built on the hypotenuse of a right-angled triangle is equal to the sum of the areas of the squares built on the cathetus*.

The teacher emphasized that the area of any regular polygon with *n* sides built on the hypotenuse of a right triangle is equivalent to the sum of the areas of homologous regular polygons, also with *n* sides, built on their cathetus. Thus, the concept of Pythagoras' Theorem generalization was presented, which with the support of GeoGebra, was not limited to just showing this relationship with the "square" polygon, but also validating it for any similar flat figure built on the hypotenuse and their respective counterparts, on the cathetus of a right-angled triangle. Mathematically, it was demonstrated that it is possible to construct, on the sides of a right-angled triangle, similar figures A_1, A_2 and A_3 so that: Area(A_3) = Area(A_2) + Area(A_1), where:

$$\frac{Área\ (A3)}{Área\ (A2)} = \frac{a^2}{b^2};\ \frac{Área\ (A3)}{Área\ (A1)} = \frac{a^2}{c^2};\ \frac{Área\ (A2)}{Área\ (A1)} = \frac{b^2}{c^2}$$

Other similar polygon models were also explored via software, such as B_1, B_2 and B_3, built on the sides of a right-angled triangle △ABC, in which their areas preserve a proportionality relationship:

$$\frac{Área\ (B3)}{Área\ (A3)} = \frac{Área\ (B2)}{Área\ (A2)} = \frac{Área\ (B1)}{Área\ (A1)} = \alpha$$

It can be written as:

$Area\ (B3) = \alpha\ .Area\ (A3)$, $Area\ (B2) = \alpha\ .Area\ (A2)$ and $Area\ (B1) = \alpha\ .Area\ (A1)$.

And from there, it was demonstrated that:

$$Area\ (B3) = \alpha\ .Area\ (A3) = \alpha\ \big(Area\ (A2) + Area\ (A1)\big)$$

$$= \alpha\ .Area\ (A2) + \alpha\ .Area\ (A1)$$

$$= Area\ (B2) + Area\ (B1)$$

completing the mathematical demonstration.

Abar (2020) explains that the use of digital technologies through software such as GeoGebra has the potential to leverage the understanding of the evolution of a mathematical object through concepts discovered, as they are researched, which we actually understand as true, given the experience in this situation.

***A posteriori* analysis and validation**

The didactic sequence was developed with the students over a period of two classes, where there was a lot of curiosity, especially in manipulating the software and the interest in learning how to construct other geometric figures and solve mathematical situations more practically. The mediating teacher initially mentioned the possibility of using GeoGebra also through the GeoGebra Suite app, available for cell phones and tablets.

The students' main difficulties identified in the internal diagnostic assessment, regarding solving questions about the Pythagoras' Theorem with manually written calculations, were: (a) identifying the sides of the right-angle triangle, that is, who are the cathetus and which is the hypotenuse, as well as (b) understand that there is no change in the measurements of the sides if the triangle is rotated and/or translated. These difficulties were not noticed by the teacher during the construction and resolution of the question via software.

In the *action situation*, students were expected to interact with the software and build the figure without major difficulties. However, we note that when constructing *n*-sided polygons on the cathetus and hypotenuse, depending on the order in which the vertices of the triangle were clicked, the polygon was constructed superimposed on the initial triangle itself, and not as an area projected onto its sides, visibly. This step needed to be explained more clearly as construction progressed.

During the *formulation situation*, the students only manually calculated the area of the polygons with $n = 3$ and $n = 4$ sides, through mental calculation, as the values were integers. Two of them had already used the software at another time and knew that there was a tool that calculated the area directly, suggesting to colleagues how to calculate the areas of polygons of $n = 5$, $n = 6$ and $n = 7$ sides. Therefore, there was no paper recording of the calculation of these areas.

For the validation situation, in fact, they calculated the sum of the areas as required in the problem and as expected in the *a priori* analysis. However, they used the calculator app, available on the Linux system of the Laboratory's computers. However, we understand that this does not invalidate your geometric learning and perception, as the most important thing was to understand the relationship between the values of the areas on each side of the constructed right-angled triangles.

At this stage, as the main conjectures presented from the dialogue between peers and interaction with the milieu, the students pointed out that:

(a) all polygons on the sides of the right-angled triangle in the construction were regular, as they had all equal sides.
(b) the areas of all polygons were different from each other.
(c) as the number *n* of sides of the polygons formed on the sides of the right triangle increased, the areas also increased proportionally.
(d) finally, they pointed out the relationship between these areas, inferring the generalization of the Pythagorean Theorem, as expected.

During the discussion, in *institutionalization situation* of knowledge, the teacher made it clear that, if there are similar particular figures A_1, A_2 and A_3 constructed respectively on the sides of a right-angled triangle, satisfying the condition: $\text{Area}(A_3) = \text{Area}(A_2) + \text{Area}(A_1)$, then any other similar figures B_3, B_2 and B_1 built on the hypotenuse and the cathetus have the same relationship, thus describing the Generalization of Pythagoras' Theorem. The teacher also took the opportunity to show this by creating a semicircle on the sides of the triangle, calculating its area via software, and concluding that the validity of this generalization was valid for any figure that configured a closed region, including non-polygonal figures.

We suggest replicating this study in different contexts within Basic Education. For example, with the use of contextualized situations, the use of GeoGebra Suite for smartphones and tablets and the construction of GeoGebra linked to Google Classroom to obtain feedback from students. We also suggest that teachers check the results of this study and adapt them to their work context and the intended audience. We believe that the curiosity aroused by the student's use of GeoGebra can encourage them to study mathematics and learn with meaning.

Final considerations

The teaching of geometric concepts in practice combined with the use of GeoGebra positively impacted the teacher's activity and student learning. GeoGebra enabled the dynamic visualization of the theme, which instigated the students' geometric perception and the relationship between the elements of the algebra and 2D construction windows.

For teaching geometry, we understand that visualization is essential so that students are directed to assimilate situations, replicate, and generalize them, in a process of searching for knowledge. By presenting a graphical, tabular, and algebraic interface for mathematical objects, GeoGebra has become an important tool for teaching and demonstrating the Generalization of Pythagoras' Theorem. In this sense, we understand that the objective of the didactic sequence was achieved and we suggest the replication of this activity model as a methodological proposal for the teacher, leading the student to participate more actively, to reflect and develop their autonomy, being a protagonist in the construction of their own knowledge, as recommended by the Theory of Didactic Situations.

The Theory of Didactic Situations associated with the use of GeoGebra software enabled the organization of a *milieu* that instigated the development of geometric perceptions autonomously, as students investigated means and possibilities to solve the problem presented, and even other unforeseen resources immediately. Didactic Engineering, as a methodology, allowed the prior organization of a didactic sequence, as well as the possible variables involved in the course of its practical implementation. Based on the course of the DE developed, it was possible to examine the software's contributions to overcoming students' learning obstacles, in addition to the possibility of internally validating the implemented teaching proposal.

Regarding the limitations of this study, we had difficulty to find other methodological proposals or questions in the textbook adopted at our school on the generalization of Pythagoras' Theorem, especially teaching suggestions with a technological bias. We also understand how difficult it is to replicate the model for a complete class of students in a conventional class at regular hours, given the precarious conditions of the schools' IT Laboratory, with an insufficient number of computers in perfect working order, as well as their physical capacity.

As a future perspective, we hope that this experience will be appreciated, disseminated, and replicated in other classrooms and other social realities, aiming to minimize barriers in learning Geometry and this topic in particular, as well as advocating what is proposed in the BNCC on approach to Geometry associated with technology.

Acknowledgements

The authors are grateful for the encouragement and financial contribution of the National Council for Scientific and Technological Development (CNPq) for the development of this research in Brazil.

Conflict of interests

The authors declare no conflict of interest.

References

Abar, C. A. A. (2020) A Transposição Didática na criação de estratégias para a utilização do GeoGebra. *Revista do Instituto GeoGebra de São Paulo, 9*(1), 59-75. https://doi.org/10.23925/2237-9657.2020.v9i1p59-75

Almouloud, S. *Fundamentos da Didática da Matemática.* UFPA, 2007.

Alves, F. R. V., & Borges Neto, H. (2012). Engenharia Didática para a exploração didática da tecnologia no ensino no caso da regra de L'Hôpital. *Educação Matemática Pesquisa*, *14*(2), 337-367. https://revistas.pucsp.br/emp/article/view/9445

Artigue, M. (1996). Engenharia Didática. In: Brun, J. *Didáctica das Matemáticas*. Instituto Piaget. Horizontes Pedagógicos, (pp.193-217).

Brasil. (2018). *Base Nacional Comum Curricular.* Ministério da Educação.

Brousseau, G. (2002). *Theory of Didactical Situations in Mathematics*. Didactique des Mathématiques, 1970-1990. Kluwer Academic Publishers.

Brousseau, G. (2008). *Introdução ao estudo das situações didáticas:* conteúdos e métodos de ensino. Ática.

Cruz, A. M. (2015). *Uma abordagem didática para o Teorema de Pitágoras*. Instituto de Matemática. Três Passos.

Leivas, J. C. P., & Oliveira, M. T. (2017). Visualização e Representação Geométrica com suporte na Teoria de Van Hiele. *Ciência & Natura*, *39*(1), 108-117. https://doi.org/10.5902/2179460X23170

Machado, P. F. (2012). *Fundamentos de Geometria Plana.* CAED.

Moran, J. M. (2007). *A Educação que desejamos: novos desafios e como chegar lá.* Papirus.

Pereira, M. G. G., Couto, A. P. N. P., & Costa, A. C. (2016). Análise de erros em questões de Teorema de Pitágoras: um estudo com alunos do ensino fundamental. In: *XII Encontro Nacional de Educação Matemática (ENEM).* São Paulo. https://periodicos.unespar.edu.br/index.php/rpem/article/view/6712

Santos, M. G. M., & Alves, F. R. V. (2022). A Engenharia Didática e a Teoria das Situações Didáticas em uma proposta didática para o ensino de Geometria Plana . *Revista Binacional Brasil-Argentina: Diálogo Entre As Ciências*, *11*(02), 98-111. https://doi.org/10.22481/rbba.v11i02.11108

Sousa, R. T., Azevedo, I. F., & Alves. F. R. V. (2021). O GeoGebra 3D no estudo de projeções ortogonais amparado pela Teoria das Situações Didáticas. *JIEEM*, *14*(1), 92-98. https://doi.org/10.17921/2176-5634.2021v14n1p92-98

Sousa, R. T., Azevedo, I. F., Lima, F. D. S., & Alves, F. R. V. (2021). Transposição Didática com aporte do GeoGebra na passagem da Geometria Plana para a Geometria Espacial *Revista Ibero-Americana de Humanidades, Ciências e Educação*, *7*(5), 106–124. https://doi.org/10.51891/rease.v7i5.1177