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Software GeoGebra as a Proposal for Mathematical Modeling Problems

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Abstract

Purpose: This study arises from an investigation into the difficulties in understanding Geometry and the mishaps in the interpretation of problems from the Mathematics Olympiads. The objective of this work is to present a didactic proposal that explores the resolution of Olympic issues, developing visualization skills and geometric reasoning with the support of GeoGebra software. **Methodology:** For this, we used Didactic Engineering as a methodology, in its first two phases - preliminary analysis and a priori analysis - considering that this study is part of an ongoing research. **Findings:** As a result, we present a didactic proposal that explores two questions of the International Kangaroo Mathematics, describing their resolution based on the assumptions of the Theory of Didactic Situations. **Significance:** Finally, we hope to contribute to the teaching work in this area and, subsequently, implement these didactic situations in the last two phases of Didactic Engineering, carrying out their experimentation, posterior analysis, and validation.

Keywords: geogebra, geometry, mathematics olympiads, mathematics teaching, mathematical modeling.

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Introduction

The guidelines for teaching Geometry emphasize the development of skills and abilities related to geometric visualization, formulation of logical hypotheses and understanding of the properties of figures. The National Common Curricular Base (BNCC) (Brasil, 2018) substantiates the importance of visualization, as a necessary skill for the conception of conjectures and the development of techniques to express notions and strategies.

Based on this premise, Sousa *et al*. (2021) mention that the GeoGebra software as a technological resource brings a set of potentialities that can support the teaching practice, helping especially with regard to the presentation of Geometry subjects considered of complex assimilation, facilitating the understanding of the student through geometric visualization.

In addition to the use of GeoGebra, we also emphasize mathematical modeling and its relevance in Geometry issues, to enable the student's understanding of the topics covered. Dias (2007) states that the use of geometric models stimulates the student's interest in the subject, through meaningful activities, instigating their curiosity about their functioning and applications in Geometry.

As teachers, we commonly perceive that students face obstacles in understanding geometry, with regard to the elaboration of geometric reasoning, showing gaps in the assimilation of theorems and axioms, as well as the relationship between them. It is still common to associate the solution of problems in Geometry with the use of ready-made formulas and algorithms, which may not actually stimulate the evolution of the student's geometric thinking from a visual understanding (Settimy & Bairral, 2020).

The Mathematics Kangaroo contest, which has been widespread in public and private schools in Brazil, as Moreira (2019) points out, commonly uses questions in its tests that involve more than the systematic use of formulas for solving problems, particularly in area of geometry. This reinforces the importance of understanding and developing geometric skills, emphasizing visualization and theorem generalization.

Thinking about the difficulties in Geometry and the models of problems suggested by the Mathematics Kangaroo, the objective of this work is to present a didactic proposal that explores the resolution of two questions of this contest, from the development of visualization skills and geometric reasoning with support of the software GeoGebra.

To achieve the objectives of this work, we used Didactic Engineering (DE) as a methodology to guide the research path, in its first two phases - preliminary analysis and a priori analysis - considering the fact that this work consists of ongoing research. DE was chosen because, according to Alves and Dias (2019), it is a methodology that brings an alternative from a systematic perspective of preparation, design, planning, modeling and allows the execution and replication of structured teaching sequences.

In the following sections we present the preliminary analyzes of this work, addressing the difficulties regarding geometric visualization, mathematical modeling, and its use with the GeoGebra software and the a priori analysis with the didactic proposal of this work, as well as the authors' considerations.

Materials and methods

Didactic Engineering (DE) comes from French Mathematics Didactics studies and consists of a methodology that guides teaching practice, because according to Artigue (1996), DE is configured in an experimental scheme, which is based on didactic achievements in the classroom, that is, the design, implementation, observation, and analysis of teaching sequences.

Systematically following this methodology in its planning and execution, we have a route guided by the following phases: i) Preliminary analysis, ii) Conception and a priori analysis, iii) Experimentation and iv) A posteriori analysis and validation. In the case of this work, as it is an ongoing research and has the character of a didactic proposal, we used only the first two phases, described briefly below.

In the preliminary analyses, we briefly describe the importance of the Olympics in school, the mishaps in geometric visualization and consequent understanding of Geometry in the school environment and mathematical modeling with the contribution of GeoGebra software, consolidating a theoretical basis for this work.

In the *a priori* analysis, we bring a didactic proposal for the teaching of Geometry with two questions from the Kangaroo Mathematics contest, solved with the help of GeoGebra, seeking to provide methodological support to the mathematics teacher.

Preliminar Analysis

Olympic-level competitions in Mathematics provide greater engagement of students in the discipline, generating growth in their learning and an improvement in their ability to solve problems. In addition, there is a motivation on the part of the student to participate, as the problems are presented in an exciting way for the students, addressing multiple mathematical, social, and cultural aspects, which causes a movement in the school, which encourages their involvement (Azevedo, 2020; Santiago, 2021).

However, as Azevedo (2020) points out, there are still many barriers for Mathematics teachers in Brazil, arising from their initial and/or continuing training with regard to working with Olympic problems in the classroom, whether in the specifically mathematical aspect, or in the pedagogical and didactic, which motivates us to carry out research on this topic.

When it comes to the field of Geometry, mathematical thinking is related to visualization, manipulation of objects and understanding of the space that surrounds us, being a field of mathematics in which students also have blockages. Settimy and Bairral (2020) state that visual thinking, which is characteristic of the study of geometry, needs to be stimulated as much as algebraic thinking. The same authors also reinforce that the prioritization of Algebra to the detriment of Geometry reverberated in the development of only one category of mathematical thinking, which shows the need to instigate ways of understanding geometric thinking in Mathematics classes.

Sousa *et al*. (2021a) explain that many difficulties in understanding Spatial Geometry by students stem from learning gaps in Plane Geometry. “The figure sketched on paper or on the board does not correspond to its original form, making it difficult to develop the mental image of the object and making it impossible to visualize and apprehend knowledge” (Sousa *et al*., 2021, p. 111). Indeed, we understand that there is a need to develop the student's geometric thinking based on visual perception.

Seeking better ways of working Geometry and stimulating the student's reasoning in this area, we propose in this work the mathematical modeling of questions at the Olympic level with the GeoGebra software. In this sense, modeling presents a consolidated direction in the process of demonstrating mathematical constructions, giving meaning to problem solving.

Blum and Niss (1991) and Bassanezi (2002), describe that mathematical modeling is related to the art of transforming reality problems into mathematical problems and solving them, interpreting their solutions in the language of the real world. According to the authors, modeling is similar to art, as it is interpreted through creative action and not through the realization of a theory or practical method, which we can link to the understanding of Olympic problems.

Alves (2019) brings a proposal in which we can infer that geometric conceptions can be developed considering the student's level of geometric knowledge, and that the construction of mathematical models developed in Dynamic Geometry software, such as GeoGebra, can support and generate a consequent learning.

Based on the above, in the following section we outline the a priori analysis of this work, considering the mathematical modeling associated with GeoGebra for working with Geometry problems.

Results and Discussions

As a partial result of this work, we bring in this section the second phase of Didactic Engineering, which consists of the conception and a priori analysis of two Olympic questions, extracted from the Kangaroo Mathematics competition, structured in a teaching sequence format, in order to offer a support to the professor of this discipline.

A Priori Analysis

In this stage of the DE, we present a didactic proposal for teaching volumes and proportions using GeoGebra 3D and its visual/manipulable resources, in which we assume the premise that the visual exploration and algebraic/geometric manipulation of this construction is designed as a guiding element in the teacher's didactic mediation when working with this theme. According to Alves (2020, p. 340) “the teacher will be able to enhance the role of visualization, through the exploration of the GeoGebra software, with a view to acquiring a mathematical culture and the outlining of intellectual habits applicable in other situations”.

In order to work in an articulated way with DE in this article, we bring the Theory of Didactic Situations (TSD) (Brousseau, 2008) as a way to organize and model a didactic situation involving the subject, based on a construction elaborated in GeoGebra, seeking to predict behaviors of the student through the proposed situation.

Brousseau (2008, p. 20) defines that “a 'situation' is a model of interaction between a subject and a specific environment”. Based on this premise, the term “didactic situations” refers to the models that describe the relations of activities between student, teacher, and the *milieu*. In summary, the TSD prioritizes the student's development in an active and autonomous way and can be modeled by phases or dialectics according to Brousseau (2008), they are action, formulation, validation, and institutionalization, exemplified throughout this didactic proposal.

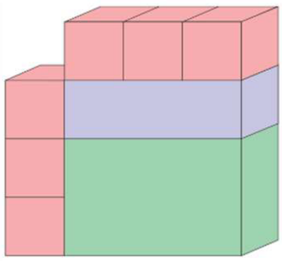
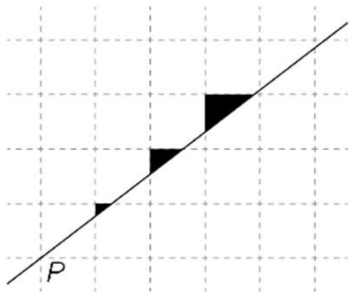
According to Brousseau (2002), the conception, organization, and planning of a didactic situation by itself demands stages in which the student is alone facing the problem and tries to solve it without the direct intervention of the teacher. This situation is called by the author as an adidactic situation, in which the student, when interacting with the proposed problem-situation, manages to solve it, without any help or direct response given by the teacher, doing so only based on their previous knowledge and experiences. It is worth emphasizing that the didactic situations are designed so that they coexist with the didactic situations, characterizing and obeying a didactic process predetermined by objectives, methods, resources, and concepts.

Initially, a *didactic contract*[1] must be established (Brousseau, 2008) between the teacher and the class, as a way of guiding the teaching and learning process. Thus, it is expected that the student, encouraged to build knowledge and take possession of knowledge individually without the direct interference of the teacher, has the perception and development of mathematical thinking through the incentives promoted by the didactic situation.

We used as material for the elaboration of didactic situations two questions from the Kangaroo Mathematics test of the year 2020, level S (student) aimed at High School, to be proposed to students and developed from the TSD dialectics, which deal with the area of prisms and proportions in geometry. In Table 1, we have the selected questions:

Table 1

Kangaroo Questions of Mathematics Brazil, year 2020, Level S (student).

Question 13	Question 22
Zilda will use six identical cubes and two different rectangular blocks to form the structure on the side, with eight faces. Before gluing the pieces together, she's going to paint each one completely, and she's calculated that she'll need 18 liters of paint (the color doesn't matter). How many liters of paint would she use if she painted the entire structure only after the parts were glued together?  (A) 8,4 (B) 9,6 (C) 11,5 (D) 12,8 (E) 16,0	A kangaroo draws a line passing through point P on the grid and then paints three triangles black as shown in the figure. The areas of these triangles are proportional to which numbers?  (A) 1 : 4 : 9 (B) 1 : 2 : 9 (C) 1 : 3 : 9 (D) 1 : 2 : 3 (E) 2 : 3 : 4

Next, we describe, based on the TSD dialectics, the development of each of these didactic situations.

Didactic Situation 1

In the situation of action, the students, in possession of the proposed problem, must carry out a careful reading and seek in their previous knowledge the concept of area of a polygon. Initially, the student will visualize the six cubes of equal measurements and the other two figures in the form of parallelepipeds, with different dimensions. It is expected that they will be able to infer that as the dimensions of an object change, its shape also changes and, consequently, its area changes.

[1] According to Brousseau (2008), the didactic contract is a set of reciprocal and expected actions from both the teacher and the class, which are pre-established so that the functioning of a didactic situation occurs efficiently.

In Figure 1, we show a sketch that the teacher can use to develop this situation, using GeoGebra's 2D and 3D windows and their manipulation:

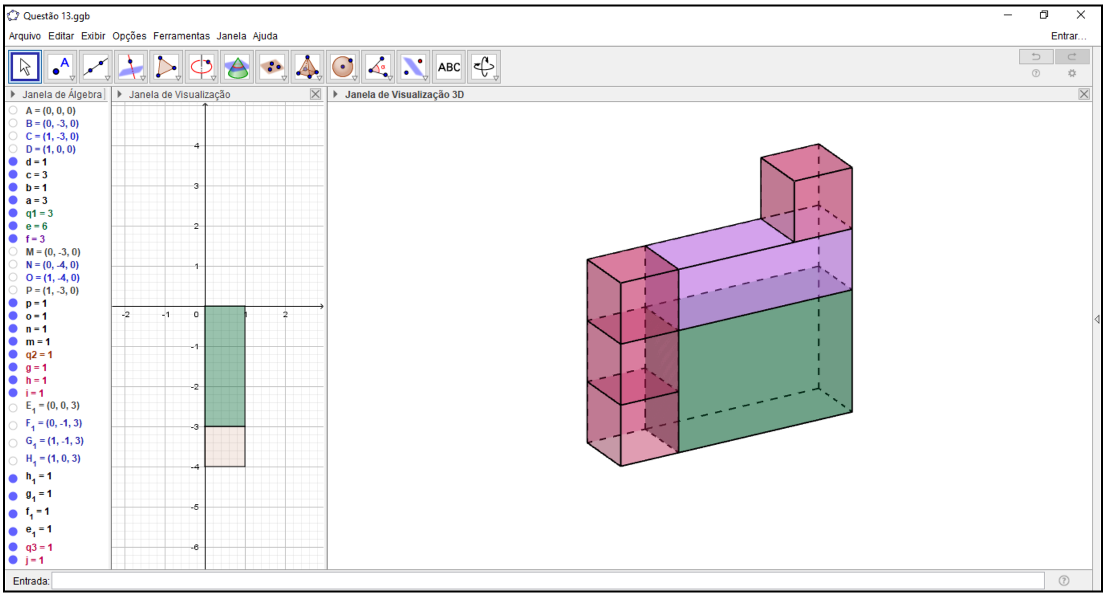


Figure 1. Visualization of 13rd question of Kangaroo Mathematical Brazil in 2D/3D format.

In the situation of formulation, students can conjecture ideas and structure their strategies for solving the question. The teacher can provide an access link to the constructed mathematical objects, making this dialectical movement possible. Thus, the student will be able to manipulate and structure a mathematical model that relates the unitary faces of each cube and the faces of the two parallelepipeds.

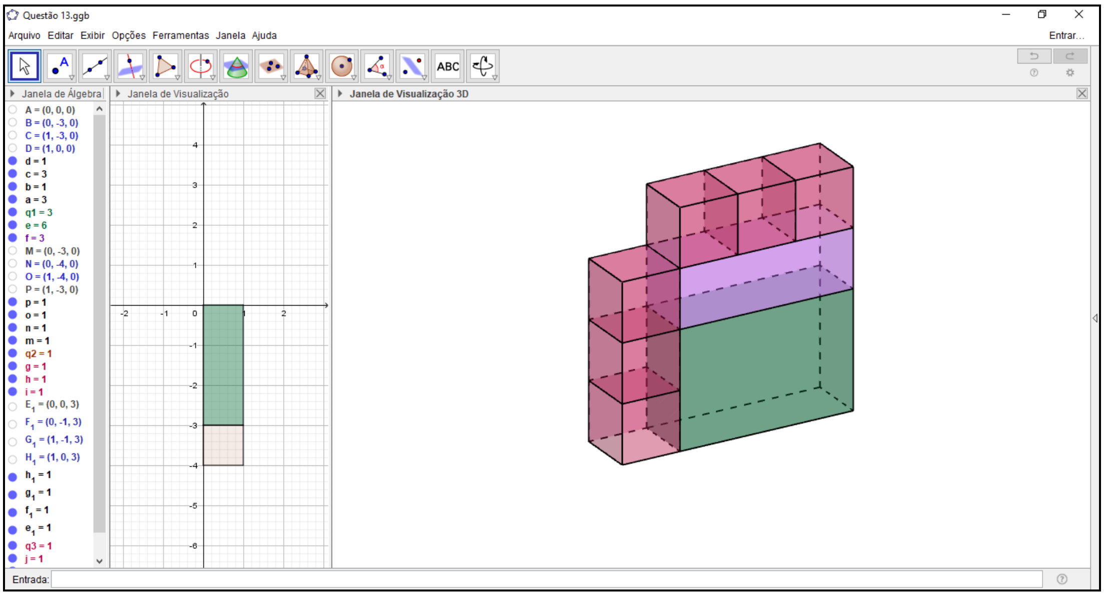


Figure 2. Visualization of 13rd question of Kangaroo Mathematical Brazil in 2D/3D format.

In the situation of validation, we expect the students to show their reasoning in an organized way, using the concept of area and comparing the dimensions of the different parallelepipeds that make up the objects. Thus, we aim for students to look for the number of faces of each mathematical object from the construction in the GeoGebra software, finding 6 x 6 + 14 + 22 = 72 faces.

At this stage, it is necessary for the student to prove what was conjectured in the previous stage, so it is expected that they present this reasoning clearly and with the help of software manipulation.

In the situation of institutionalization, the teacher starts to intervene, analyzing the speeches and arguments presented by the students, discarding erroneous conceptions and inadequate mental models, and formalizing the mathematical subject worked. Thus, the teacher can present the stages of the solution, comparing and reviewing the concept of area of a prism using Leonardo (2016), who brings that the area of a prism is the numerical value that represents its surface, being represented by the sum of the area of its two bases and the lateral area, that is, $A_{total} = A_{lateral} + 2.A_{basis}$.

Since the construction gives the value of 72 (unit faces) divided by 18 (paint liters) = 4, we conclude that one liter of paint paints four unit faces. In this way, when structuring the solution, we have: 2 faces composed of 15 single faces; 2 faces by 4 single faces; 2 faces by 3 single faces, and; 2 faces by 1 single face, totaling a value of 2 x (15 + 4 + 3 + 1) = 46. So, the total liters used to paint it will be 46/4 = 11.5 liters (alternative C).

In the GeoGebra visualization window on the left side, we can observe the resolution of the problem with the presentation of each object, using the show/hide command, as shown in Figure 3:

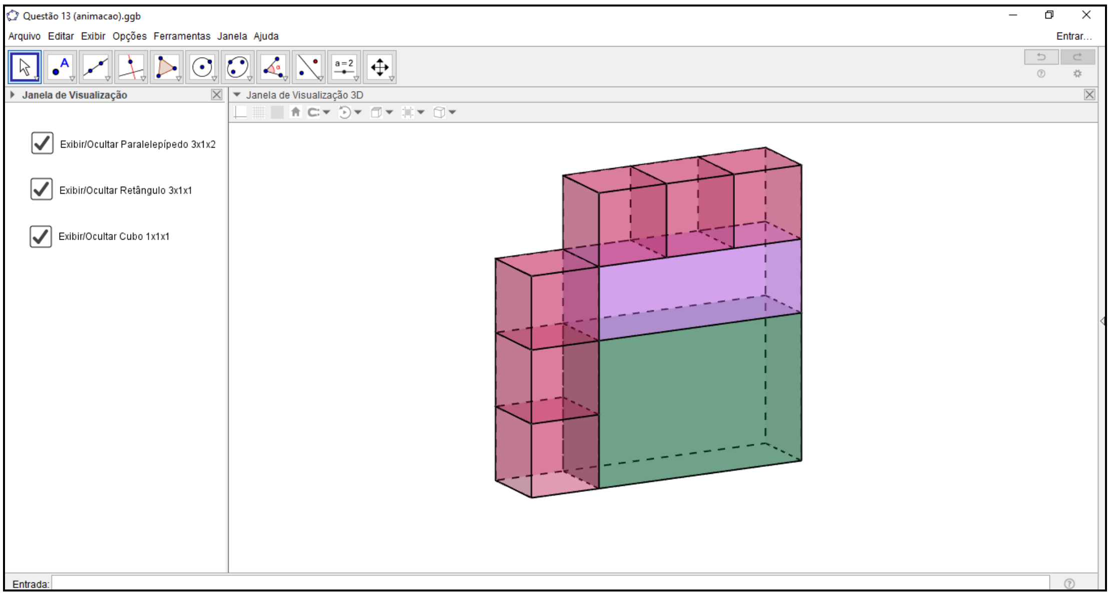


Figure 3. 2D/3D visualization provided by GeoGebra software with all objects.

Didactic Situation 2

In the situation of action, the student must observe, after careful reading, all the possibilities for structuring the solution to the problem, using the dynamism of the GeoGebra software. The statement of this didactic situation brings a proportion relationship in its alternatives, in which initially the student can notice the similarity between the triangles. In Figure 4 we have the representation of this situation in GeoGebra:

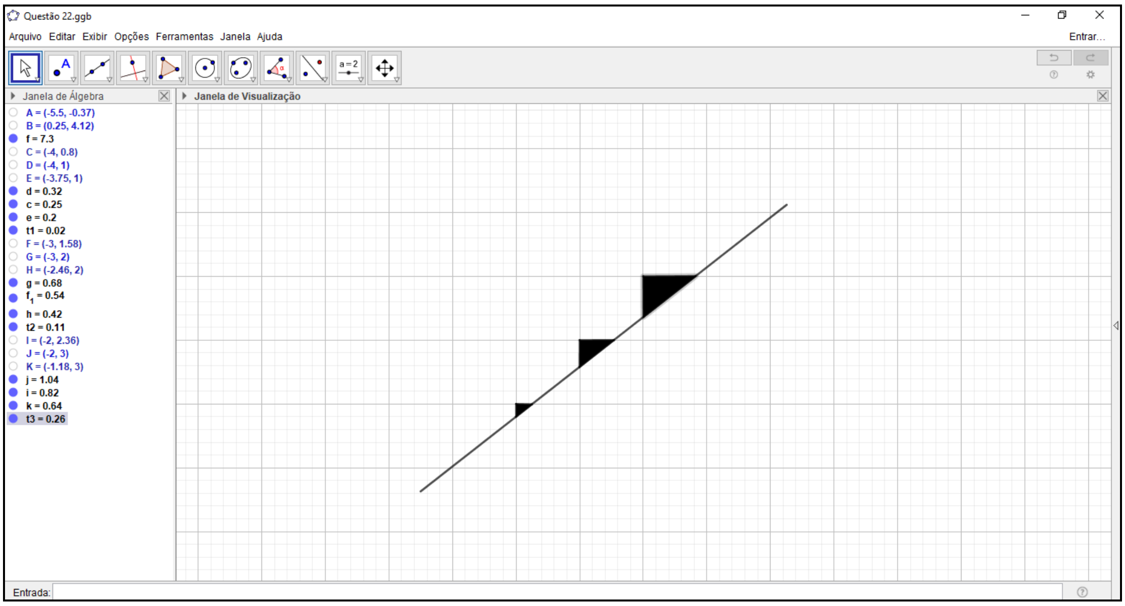


Figure 4. Construction of the second didactic situation in the GeoGebra software.

In the situation of formulation, students in possession of the construction should be encouraged to manipulate the construction and record their observations. Note, in the construction of Figure 5, an example of movement that can stimulate geometric thinking, with the modification of the steps and the proportion of their sides measuring 1, 2 and 3, respectively:

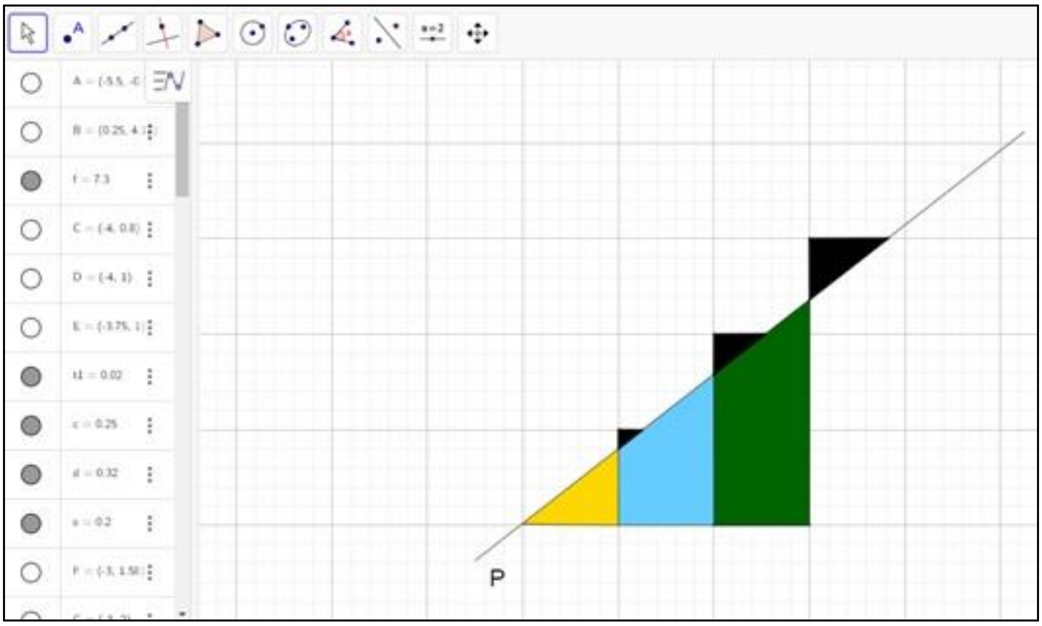


Figure 5. 2D/3D geometric object provided by GeoGebra software.

It is also important to point out that, when removing each colored figure, the student is able to notice the proportion being built with the display/hide command in the mathematical object. Thus, to solve the problem, the student must handle only the colored figures.

In the validation situation, the student is expected to formulate resolution sequences from the observations made to arrive at the construction of a mathematical model, as shown in Figure 6:

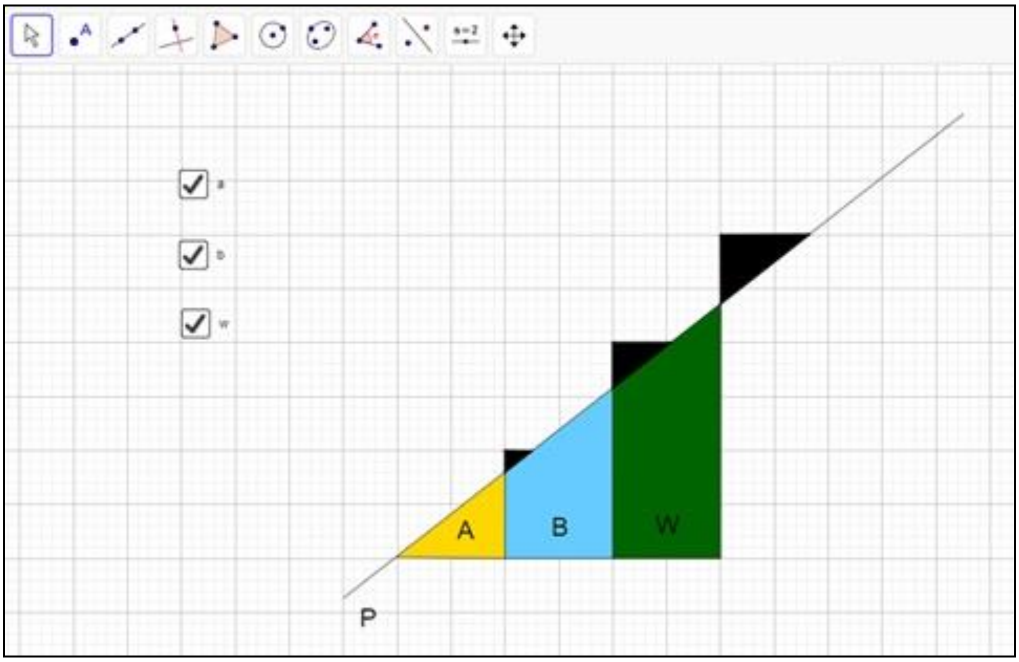


Figure 6. Construction of the question in the GeoGebra software.

At this time, students are expected to use the display/hide function in the selection boxes (a, b, w), and develop their geometric reasoning based on the visualization of the figure.

In the situation of institutionalization, the teacher resumes the previous steps, formalizing the mathematical concept of proportion associated with geometric knowledge. The teacher can use the solution provided by Kangaroo de Mathematical Brazil (2020, p. 123), which explains that the black triangles are similar to the complementary triangles highlighted in the figure (overlapping colors) and their sides measure 1, 2 and 3 respectively. Then the black triangles are similar to each other, keeping the similarity ratios. Therefore, the areas of these triangles are in the same ratio as the squares of the similarity ratios, that is, 1: 4: 9 (alternative A).

Conclusions

The conception of this work arose in the face of the difficulties related to the teaching of Geometry, as a way of helping both the teacher in his practice – considering the fact that the blackboard and brush are not always enough to clearly explain some situations in Geometry –, as well as the student in the development of his logical geometric thinking.

Thus, this work proposes the use of mathematical modeling in two problem situations involving Geometry, extracted from the Kangaroo Mathematics contest, aiming to explore the geometric visualization with the contribution of the GeoGebra software, showing different ways of working with materials from the Olympics in the classroom. classroom, via technological resources.

Based on the aforementioned theoretical framework, we realize that many of the students' difficulties in Geometry are due to the way the subject is approached in the classroom, in a traditional, mechanized way and with little visual exploration. Visualization in Geometry, from different perspectives, can develop the student's thinking to understand the world around him, as well as develop in other areas of knowledge.

As future perspectives for this study, we intend to implement these didactic situations, as well as elaborate others, to collect data and execute the last two phases of Didactic Engineering - experimentation and a posteriori analysis and validation - verifying the validity of what was conjectured in this a priori analysis in the form of a didactic proposal.

We hope that this work can contribute to the teaching of Geometry, as a support to the mathematics teacher in the development of his teaching work, as well as to a gradual progress of the student, with regard to geometric thinking from the combination of mathematical modeling, Geometry and GeoGebra.

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Conflict of interests

The authors declare no conflict of interest.

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