COOLING HOT COFFEE USING NONLINEAR EQUATION NUMERICAL METHODS

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ABSTRACTS

Freshly brewed hot coffee requires time to reach the ideal temperature for comfortable consumption, typically around 58–66 °C. Consumers generally rely on estimation or experience when waiting, without a precise scientific approach. This study examines the cooling process of coffee as a heat transfer phenomenon that can be modeled using Newton's Law of Cooling, in which the rate of temperature decrease is proportional to the temperature difference between the object and its environment. Since the governing equation is nonlinear, solving it requires a numerical method. The Newton-Raphson method was selected for its efficiency in solving single-variable nonlinear equations and its fast convergence. The simulation was conducted using Python, with the following parameters: an initial temperature of 80.57 °C, ambient temperature of 27 °C, target consumption temperature of 62.99 °C, and a cooling constant of 22.24×10^{-3} s^{-1} based on previous experimental data. The results showed that the ideal consumption temperature is reached in approximately 17.88 minutes. The iteration graph demonstrated a rapid decrease in function values, requiring only four iterations to converge. While the simulation showed high accuracy during the initial cooling phase, minor deviations occurred as the temperature approached ambient levels. This discrepancy is likely due to the assumption of a constant cooling coefficient, whereas in reality it may vary depending on the solution temperature and air convection conditions. This model can be used to objectively predict the optimal time to enjoy coffee based on scientific calculations. In addition to benefiting home consumers seeking the best coffee-drinking experience, the findings of this research can be applied in the culinary and hospitality industries to consistently and efficiently serve hot beverages at the right temperature. By providing predicted consumption times, the risk of burns from excessively hot drinks can be minimized, thereby enhancing customer satisfaction. This approach can also be adapted for other hot beverages requiring temperature control, offering broader applicability in the fields of food engineering and thermal design.

Keywords: Coffee; Temperature; Numerical method.

INTRODUCTION

Coffee is a beverage derived from the processing and extraction of beans from the coffee plant (Gita Arumsari et al., 2021). It is also one of the most popular agricultural commodities worldwide, consumed in nearly every country (Salsabila et al., 2021). Today, drinking coffee is no longer merely about satisfying taste preferences it has become a lifestyle, particularly among urban communities and millennial generations (Solikatun et al., 2015; Putri, 2019). Coffee has become a lifestyle beverage for a portion of the Indonesian population (Yusuf Indriyanto & Basar Maringan Hutauruk, 2023).

Coffee is typically enjoyed at an ideal temperature of around 58 °C–66 °C (Ristenpart et al., 2022). While the recommended serving temperature for hot beverages is often around 85 °C, this can pose a burn risk to consumers (Adhikari et al., 2019). Therefore, knowing when coffee reaches a safe and comfortable drinking temperature becomes important, especially from a scientific standpoint. In practice, people often wait for their coffee to cool based on guesswork, which tends to be inefficient and subjective.

Most previous studies have focused more on experimental measurements than on mathematical modeling. However, this cooling process can be modeled using the principles of natural convection heat transfer, where the temperature decrease is proportional to the difference between the object's temperature and the surrounding environment (Sumardi Hadi Sumarlan et al., 2023). The equation involved is nonlinear, requiring the use of numerical methods such as Newton-Raphson, which is known for its efficiency and fast convergence (I Wayan Sudiarta, 2019). This approach is also applied in various thermodynamic studies, such as modeling the efficiency of heat engines at maximum power conditions, where Newton's Law of Cooling serves as a foundational reference (Yan & Guo, 2012). This study utilizes the Newton-Raphson method to determine the optimal coffee consumption time based on previous experimental data.

The aim of this study is to identify the ideal consumption temperature of coffee by modeling the temperature drop from a hot state to a comfortably drinkable level. Through this approach, coffee enthusiasts can determine the best time to enjoy their beverage without relying on trial and error. Additionally, the findings are expected to serve as practical guidelines for culinary professionals in serving coffee at an optimal temperature.

This study used a 3-in-1 instant coffee solution with a concentration of 6%, dissolved in 150 mL of boiling water. This solution was selected as it closely resembles the common way coffee is consumed by the general public. Experimental temperature data were obtained from the study by Ohoiwutun et al. (2023), which previously investigated the cooling rate of coffee using an LM35 sensor and an Arduino microcontroller. This reference served as the basis for validating the mathematical model in the numerical simulation. The hardware from the prior study and Python software version 3.x in the current research were integrated to quantitatively and systematically represent the cooling process. The use of experimental data also provided a strong foundation for testing the simulation model's accuracy.

The coffee cooling model is based on Newton's Law of Cooling, formulated in exponential form. The simulation process began with modeling the temperature as a function of time, which was then analytically differentiated to obtain the first derivative required for the Newton-Raphson method. The nonlinear equation was solved iteratively using the Newton-Raphson method, starting with an initial time guess $t0$, until the root approached a predefined error tolerance. To ensure result stability, simulations were run multiple times with variations in the initial guess. This method was chosen due to its high convergence rate and good efficiency in solving single-variable nonlinear equations, as demonstrated by Chan et al. (2025) in their implementation using Python and the Autograd library.

All key parameters in the simulation initial temperature of 80.57 °C, ambient temperature of 27 °C, target temperature of 62.99 °C, and cooling constant of 22.24×10^{-3} s^{-1} were determined based on previous experimental results for the 6% coffee solution. The simulation was conducted using the Google Colab platform for seamless Python code integration and result visualization. Additionally, cross-validation was performed between the simulation outcomes and experimental values to assess the model's accuracy. The simulation results, presented in graphical and tabular forms, were used to evaluate the performance of the Newton-Raphson method, particularly in the context of everyday thermal phenomena such as coffee cooling. With its systematic methodological structure, this approach can be widely applied to various other hot beverage cooling scenarios.

METHODS

The numerical simulation in this study was based on experimental data obtained from the study by Ohoiwutun et al. (2023), in which coffee temperature was measured using an LM35 sensor connected to an Arduino microcontroller. The data included the initial temperature of the coffee after brewing, ambient temperature, target consumption temperature, and the cooling constant calculated from the exponential cooling curve of a 6% concentration 3-in-1 coffee solution. All of these parameter values were used as inputs for the modeling and simulation validation process. By utilizing secondary data from previous experiments, this simulation did not involve direct temperature measurements but instead mathematically reconstructed the coffee cooling process to evaluate the model’s accuracy against real-world data.

The following figure shows the flowchart of the numerical simulation process using the Newton-Raphson method:

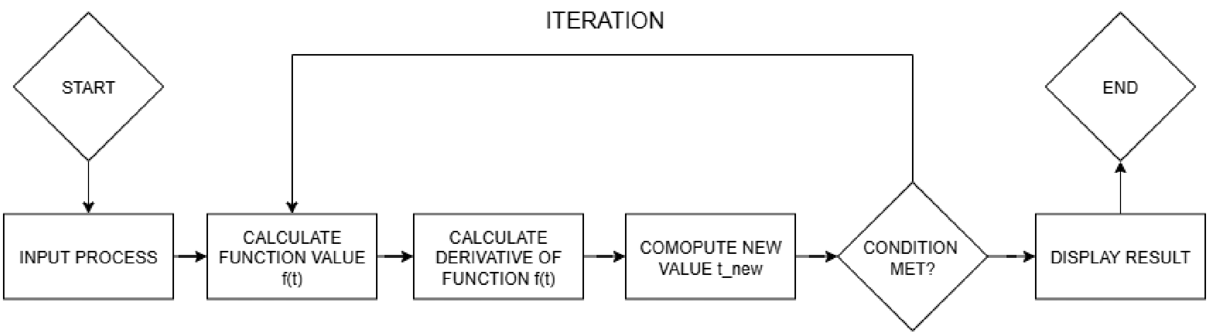

Figure 1. Flowchart of the Coffee Cooling Simulation Process Using the Newton-Raphson Method

The flowchart in Figure 1 illustrates the sequence of the numerical simulation using the Newton-Raphson method to determine the time required for coffee to reach the ideal consumption temperature. This method was chosen due to its high convergence rate, even in cases involving nonlinear functions with a single root, as explained by Akram & ul Ann (2015), who demonstrated that Newton-Raphson is faster and more efficient than other numerical methods. Additionally, a study by (Rasheed et al., 2021) showed that the method can be modified to improve accuracy and stability when solving nonlinear equations, especially in experimental contexts. The process begins by inputting initial parameters such as coffee temperature, ambient temperature, target temperature, cooling constant, and an initial time guess. The system then calculates the value of the function $f(t)$ and its derivative $f'(t)$, followed by updating the time value t using the Newton-Raphson formula. Iteration continues until the difference in the function value approaches zero or falls within a specified tolerance range. If convergence is achieved, the result representing the optimal time is displayed, and the program ends. To better understand the basis of this calculation, the following section presents the mathematical equations used in the simulation.

The coffee cooling process is modeled using Newton's Law of Cooling, with the exponential equation as follows:

$$T(t) = T_{\text{env}} + (T_0 - T_{\text{env}}) \cdot e^{-kt}$$

In the equation, $T(t)$ represents the temperature of the coffee at time t, Tenv is the env temperature, $T0$ is the initial temperature of the coffee, k is the cooling constant, and e is the exponential number (the base of the natural logarithm). To determine the time required for the coffee to reach the ideal consumption temperature, the equation is transformed into a nonlinear form:

$$f(t) = T_{\text{env}} + (T_0 - T_{\text{env}}) \cdot e^{-kt} - T_{\text{target}} = 0$$

The value of t is determined using the Newton-Raphson method with the aid of a Python program. In this study, the initial coffee temperature was set at 80.57 °C, based on experimental data recorded at the 0-minute mark for a coffee solution with a 6% concentration. The Environment temperature used was 27 °C, corresponding to room conditions during the experiment. The target consumption temperature was set at 62.99 °C, referring to the upper limit of the ideal range for drinking coffee. The cooling constant k used in the simulation was 22.24×10^{-3} s^{-1}, based on experimental results from the 6% concentration coffee solution, as described in the study by Ohoiwutun et al. (2023).

Wibowo et al. (2023) showed that the heating and cooling processes of water follow an exponential pattern according to the Newtonian model, with the constant k determining the rate of temperature change. However, Telis-Romero et al. (2000) emphasized that the value of k can vary depending on the concentration and temperature of the coffee solution, and is therefore not necessarily constant throughout the cooling process. All of these parameters were used in the numerical modeling of the coffee cooling process, which is visualized in the flowchart in Figure 1. The flowchart illustrates the steps involved in the iterative Newton-Raphson method used to calculate the time required to reach the ideal consumption temperature.

RESULTS AND DISCUSSION

The simulation results show that the Newton-Raphson method successfully predicted the coffee cooling time with a rapid convergence rate. The graph illustrates a decrease in the error function value **|** *f* (*t*)**|** at each iteration, eventually reaching a tolerance level close to zero.

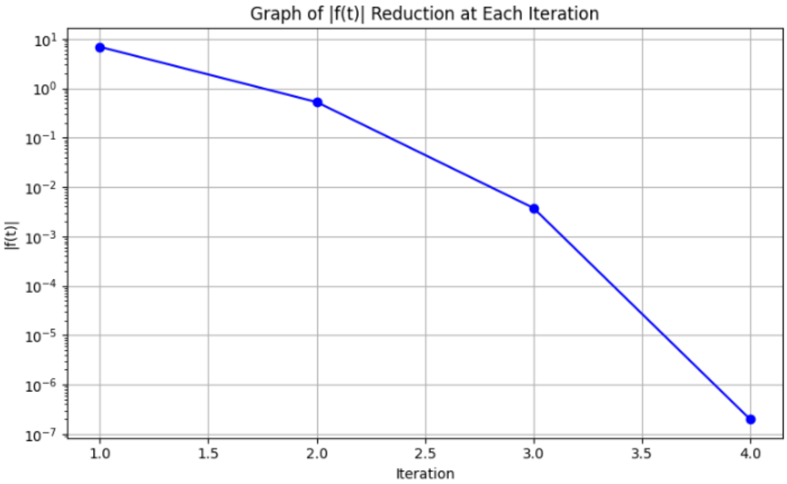

Figure 2. Graph of the Decrease in | f(t)| Value at Each Iteration

The graph shows the decrease in the absolute value of the function **|** *f* (*t*)**|** at each iteration during the process of solving the nonlinear equation using the Newton-Raphson method. It can be observed that **|** *f* (*t*)**|** significantly decreases from the first to the fourth iteration, indicating that the method converges rapidly. This logarithmic decrease pattern suggests that each iteration step consistently brings the solution closer to the root of the function, with progressively smaller error values.

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===== ITERATION DETAILS =====
Iteration 1:
  t = 10.00000 minutes
  f(t) = 6.89788
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Iteration 2:
  t = 17.23180 minutes
  f(t) = 0.52613
----------------------------
Iteration 3:
  t = 17.87964 minutes
  f(t) = 0.00377
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Iteration 4:
  t = 17.88436 minutes
  f(t) = 0.00000
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Figure 3. Detailed Values of t and f(t) at Each Iteration

This figure presents a detailed overview of the numerical calculation results, showing the values of time *t* and the function *f* (*t*) at each iteration. The initial time value was set at 10 minutes and was gradually corrected by the Newton [...] unction value reache[...] ith the convergence

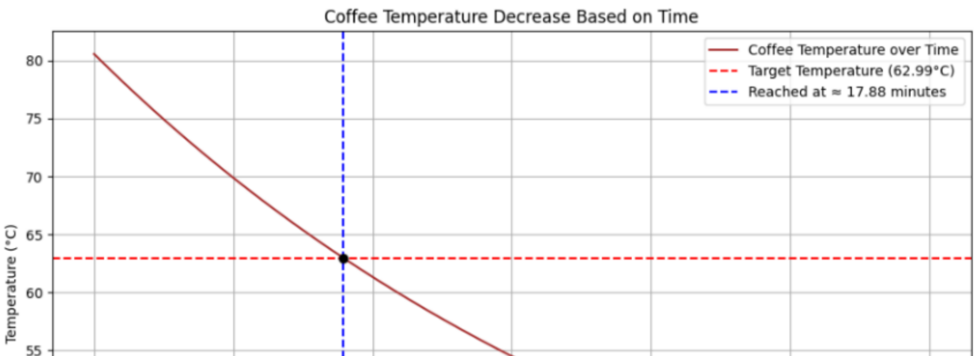

Figure 4. Coffee Temperature Decrease Graph Based on Simulation Time

This figure shows the graph of the coffee temperature decrease over time based on the numerical simulation results. The curve indicates that the coffee temperature gradually drops from the initial value of around 80 °C toward the ambient temperature, with a rapid decline at the beginning that slows over time. The horizontal line marks the target consumption temperature of 62.99 °C, while the vertical line shows that this temperature is reached approximately 17.88 minutes after brewing, representing the optimal time to enjoy the coffee.

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===== FINAL RESULT =====
Time when coffee temperature = 62.99°C:
≈ 1073.06 seconds ≈ 17.88 minutes
Number of Newton-Raphson iterations: 4
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Figure 5. Final Result of the Newton-Raphson Method Calculation

This figure presents the final result of the calculation using the Newton-Raphson method in text form. Based on the computation, the coffee temperature reaches 62.99 °C at approximately 1073 seconds or 17.88 minutes. This result was obtained through four iterations, demonstrating that the Newton-Raphson method is not only accurate but also efficient in solving the coffee cooling problem numerically.

At high temperatures (80.57 °C – 62.99 °C), the time difference between the simulation results and the experimental data is relatively small, indicating good model accuracy during the initial phase of the cooling process. However, as the temperature approaches the ambient level (below 40 °C), the simulation tends to predict a faster cooling process compared to the experimental data. This discrepancy is likely due to the assumption that the cooling constant kkk remains fixed, whereas in real conditions its value can vary depending on temperature and air convection.

Table 1. Comparison of Time to Reach Target Temperature Based on Reference Data and Numerical Simulation

Target Temperature (°C)	Reference Time (Minutes)	Simulation Time (Minutes)	Difference (Minutes)
80.57	0	0	0
71.29	8	8.55	0.55
62.99	17	17.88	-0.88
56.64	26	26.61	-0.61
52.25	34	33.82	-0.18
49.32	41	39.37	-1.63
44.92	51	49.24	-1.78
41.99	60	57.27	-2.73
39.06	72	67.05	-4.95
37.11	83	74.9	-8.02
36.18	92	84.61	-7.39
34.18	102	90.36	-11.64
32.71	112	100.66	-11.34
31.74	123	109.04	-13.96
31.25	132	113.94	-18.06
30.76	143	119.45	-23.55
30.27	154	125.73	-28.27
29.79	163	132.87	-30.13
29.3	172	141.55	-30.45
28.81	182	152.32	-29.68

Moreover, phenomena such as the Mpemba effect suggest that the cooling behavior of liquids does not always follow an ideal exponential model, meaning that Newton's Law must be understood within certain limits of validity (Pankovic & Kapor, 2012).Additionally, Telis-Romero et al. (2000) emphasized that the value of *k* may vary depending on the temperature and concentration of the solution, so Newton's exponential model may need to be adjusted under specific conditions. O'Sullivan (1990) noted that this model has limitations, particularly when evaporation, air circulation, or temperatures approaching ambient are involved. Widjaja (2010) also pointed out that interpreting the value of *k* is not always straightforward, and simulation results may diverge from observations, especially during the final phase of the cooling process. Theoretically, Nath et al. (2008) explained that a large temperature difference between the system and the environment can trigger dynamic feedback from the thermal reservoir, causing the cooling rate to be faster than predicted by the classical exponential model.

CONCLUSION

This study simulated the coffee cooling process based on Newton's Law of Cooling using a numerical approach through the Newton-Raphson method. With an initial coffee temperature of 80.57 °C and an ambient temperature of 27 °C, the simulation showed that a comfortable drinking temperature of 62.99 °C was reached in approximately 17.88 minutes, aligning with experimental data for a 6% coffee solution reported by Ohoiwutun et al. (2023). This result indicates that the rate of temperature decrease is rapid at the beginning but gradually slows down over time, especially as the coffee temperature approaches the ambient temperature. The closer it gets to the lower temperature limit, the slower the cooling process becomes due to the decreasing temperature difference between the coffee and its environment. To reach a final temperature of around 28.81 °C, it would take more than 180 minutes. This numerical approach enables scientific prediction of optimal coffee consumption time and offers practical benefits for consumers.

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