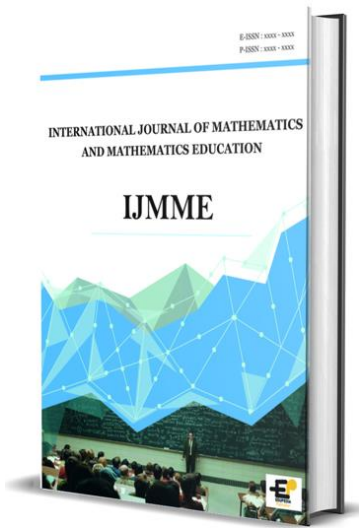

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Ida Hamidah*
STAI Al-Bahjah Cirebon, Indonesia

Susilawati
STAI Al-Bahjah Cirebon, Indonesia

Shafira Meiria Rahmasari
STAI Al-Bahjah Cirebon, Indonesia

Abdul Khalim
Beijing Jiaotong University, China

To cite this article:
Hamidah et.al. (2024). Design of Mathematical Proof-Based Logarithm Teaching Materials. *International Journal of Mathematics and Mathematics Education (IJMME), 2*(1), 15-30 https://doi.org/10.56855/ijmme.v2i1.933

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IJMME International Journal of Mathematics and Mathematics Education

February 2024, Vol. 02, No. 01,15-30 doi: 10.56855/ijmme.v2i1.933

Design of Mathematical Proof-Based Logarithm Teaching Materials

Ida Hamidah *, Susilawati Susilawati, Shafira Meiria Rahmasari, and Abdul Khalim

Article Info

Article History

Received:
November 27, 2023
Accepted:
December 24, 2023

Keywords

Teaching Materials,
ADDIE,
Mathematical Proof,
Logarithms.

Abstract

This research aims to design logarithmic teaching materials based on mathematical proof. The subjects in this study were 23 students of SMA 1 Sumberjaya, Majalengka Regency, Indonesia, consisting of 23 students in class X IPA-2. This research is a pre-research of the 4P development design study adapted from the Thiagarajan model et al., namely: (1) Definition, (2) Design, (3) Development, and (4) Disseminate. In this study, the stages carried out are limited to stage 3, namely development. The deployment phase will be carried out again later. The method used in this study is qualitative, with several data collection techniques in the form of tests of students' mathematical proving abilities in logarithmic material, interviews, and teaching material validation questionnaires. The study results showed that teaching materials were in the appropriate category with a very good predicate, with a percentage of 88.93% from material experts and 85.08% from media experts. The results of the practicality test on students amounted to 82.5%, indicating that the teaching materials made have a practicality category with a good predicate.

Introduction

Mathematics, as one of the basic sciences, both in the applied and reasoning aspects, has a very important role in improving mastery of science and technology. The higher the level of mathematics material, the greater the difficulty in learning it. Therefore, in studying mathematics, students need mathematical abilities, including proving mathematics. According to Suryadi (Suryana, 2013), mathematical proof ability is a mathematical process that is considered difficult. However, this mathematical proof ability will be very helpful in getting used to higher mathematics.

Research by Santosa (2012) states that the problem of mathematical proof still needs to be completely answered. Many other aspects and points of view can be explored and used as research material related to formal mathematical proof. Hernadi's (2008) research results concluded that learning mathematics by understanding evidence requires time to understand mathematics as a logical language. Therefore, broad mathematical insight is needed to prove more complicated facts.

According to Dumas and McCarthy (2014), most mathematicians do not think about truths related to exact sciences, and in essence, no mathematicians write formal truths in exact sciences. In other words, proof in exact sciences such as mathematics still needs to be widely discussed and researched in more depth. However, this does not mean that the ability to prove is not learned, but rather that evidence and its role have various forms depending on the level of education. As Hers (Stavrou, 2014) states, the exact definition of evidence and its role varies based on the context and level of education. Still, the general purpose of evidence is to verify, explain, communicate, and systematise reports in a deductive system.

Teaching materials have a very important role in achieving students' competencies. In teaching materials, various kinds of writing become reading material for students, so whether students are interested in learning the material and skills being taught depends on how well the teaching materials are made (Nofrianni & Syahniar, 2019). However, currently circulating teaching materials rarely discuss mathematical proof. Suandito's research (2017) revealed that students tend only to use what is written in books or what the teacher conveys as something that does not need to be proven, while the most basic motivation for why people need to prove a mathematical statement is to ensure that what is considered true is true.

Efforts to create an effective and efficient learning process must start with the way of teaching and the media teachers use. As explained in Setiawan and Basyari's research (2017), teachers should be able to realise the teaching process appropriately through effective learning and condition students in a conducive manner. Good teaching methods can help students learn and master subject matter in depth (Gilis & Winarta, 2019). Utari and Hartono (2019) said that books or teaching materials function as facilitators who can help students learn independently and develop their abilities, including their mathematical abilities. Therefore, the teaching methods and learning media must be appropriate to effectively and efficiently achieve the desired learning objectives.

There are two types of teaching materials. First, the pickled ingredients are printed. Second, non-printed teaching materials. Printed teaching materials include handouts, books, modules, worksheets, brochures, and leaflets. Meanwhile, non-print teaching materials include audio, video, learning videos, interactive multimedia, and the Internet (Setiawan & Basyari, 2017; Wanasima & Utami, 2019). Both types of teaching materials have their respective advantages and disadvantages. However, this research will print the teaching materials as printed module teaching materials. A module is an independent learning package that includes a series of learning experiences systematically planned and designed to help students achieve learning goals (Setiyadi, 2017). Learning using modules mainly aims to achieve effective and efficient learning goals.

Of the many mathematical materials, one of the materials that facilitate the development of mathematical proof skills is logarithms. Logarithms is a mathematics subject taught at the high school level. Among other things, Joyce's (2015) research found that one of the best ways to define and prove functions regarding logarithms is with calculus. Meanwhile, from research conducted by Latreuh (2014), which examined the growth behaviour of logarithmic derivatives, several relationships were

proven between the distribution of solution values for linear differential equations and the growth of logarithmic derivatives.

Balacheff (Mahfudy, 2017) said that one can only learn mathematics by learning mathematical proofs and how to make them. In addition, proof skills can improve broader mathematical skills (Firmasari & Sulaiman, 2019). Furthermore, Tall (Kartika & Yazidah, 2019) explained that in the case of university mathematics worldwide, students' construction and understanding of mathematical proofs must be emphasised. Even though proof is important, many students still need help to prove it (Maryono et al., 2018; Mahfudy, 2017; Syamsuri & Santosa, 2017). Therefore, this research aims to design teaching materials for mathematical proof to provide a simple picture and reflection regarding mathematical proof before high school students reach advanced mathematics in college.

Method

The method used in this research is a qualitative method with R & D development (Mahfudy, 2017; Syamsuri & Santosa, 2017; Hamidah, 2022). This method was chosen to make it easier and more detailed for researchers to explain social symptoms or phenomena that arise when designing teaching materials. They are difficult to express using quantitative methods.

Research subjects were taken randomly from 10 existing classes, and one class with 23 students was selected from class X Science-2 SMA Negeri 1 Sumberjaya, located in Majalengka Regency, Indonesia. This model is modified to 4D, namely defining, designing, developing, and disseminating. According to Arkadiantika et al. (2020), the 4D model is a development model that can be used to develop various types of learning media. However, this research used definition, design, and development. Several other researchers also used the same thing: limiting it to only the third stage (Setiyadi, 2017; Nofrianni & Syahniar, 2019; Kintoko & Nurrohman, 2020). The reason for using this model is that it takes a short time because the stages are relatively simple (Johan et al., 2023). In line with this, Salsabila et al. (2023) stated that the work process using a 4D model is fast but still has a complete and systematic development description that can be used to develop media and product development results that have been tested because before being distributed, product assessments are carried out by experts, namely material experts and media experts.

The definition stage includes five steps, namely: (1) curriculum analysis, (2) student analysis, (3) task analysis, (4) concept analysis, and (5) formulation of learning objectives. This definition stage is the stage of analysing the needs needed in preparing textbooks. This stage will be the initial foundation for the next stages.

Next is the design stage, consisting of three steps, namely: (1) preparing benchmark reference tests, (2) selecting appropriate media, and (3) format selection. This design stage takes the form of preparing a learning device prototype. The results of the definition stage are outlined in the design stage until a

learning tool development is developed.

The next stage is the development stage, where the completed design will be developed to produce revised learning tools based on expert input. This development stage includes (1) validation of the device by experts followed by revision, (2) simulation in the form of activities to operationalise lesson plans, and (c) limited trials with real students. The results of this stage are used as a basis for revision.

The next step is the development of the device to test the effectiveness of using the device in teaching and learning activities. However, due to the author's limitations, the research procedures were sufficient until the learning device development design was declared suitable for use by experts.

The instruments used include test and non-test instruments. The author calls the test instrument in this research the Respondent Ability Test (TKR). This TKR consists of a written test of eight questions. The test instruments used as TKR were developed based on the ability to read and construct evidence. This test instrument is used to obtain a needs analysis. The test criteria reference is based on several calculations, including the test items' validity, reliability, difficulty level and distinguishing power.

Meanwhile, the non-test instruments used were semi-structured interviews and validation questionnaires. Three material and three media expert validators assess the teaching materials' validity. This validation analysis of teaching materials refers to the validation of teaching materials, according to Akbar (2013), with the following formula.

$$V_{ah} = \frac{TSh}{TSe} \text{ x } 100\%$$

Information:
V_{ah}= expert validation
TSh = maximum expected total score
TSe = total empirical score (validation results from validators).

After each validator validates the teaching materials, the researcher carries out combined validity with the following formula:

$$V = \frac{V_{a1} + V_{a2} + V_p}{3} = \cdots \%$$

Information:
V_{a1} = First expert validation
V_{a2} = Second expert validation
V_p= User validation (teacher)
V = Combined validation

The next step after obtaining the percentage for each validated component is to determine the validity

of each validated component by referring to Table 1 below.

Table 1. Validation Criteria for Teaching Materials

Scale (%)	Instrument Validation Criteria
85,01 ≤ V ≤ 100,00	Very valid or usable without revision.
70,01 ≤ V ≤ 85,00	Fairly valid or usable but needs minor revisions.
50,01 ≤ V ≤ 70,00	It needs to be validated more and is not recommended for use because it needs major revisions.
01,00 ≤ V ≤ 50,00	Invalid, or should not be used

Source: (Akbar, 2013)

A format for evaluating the validation of teaching materials in the form of a questionnaire was created to facilitate the process of analysing the validation of teaching materials. The questionnaire created was modified according to the indicators in the research design. The validation questionnaire was distributed using a measurement scale. The measurement scale is a Likert scale modified into four options, eliminating the undecided option. The results of this validation questionnaire will later be interpreted into the Akbar formula explained above.

The next step is to assess the feasibility of a learning media for the subjects Mathematics Class X Science 2 at SMAN 1 Sumberjaya. After the data is obtained, see the weight of each response and calculate the average score with the following formula (Ernawati & Sukardiyono, 2017).

$$\bar{x} = \frac{\sum x}{n}$$

Information:

$\bar{x}$ = average score

n = number of assessors

x = total score for each

Then, for the result, the percentage formula can be calculated using the following formula.

$$Result = \frac{total\ score\ are\ obtained}{maximum\ score}\ x\ 100\%$$

The interviews were designed according to needs, while the reference criteria for validating textbook suitability can be seen in the following table (Wardathi & Pradipta, 2019).

Table 2. Textbook Eligibility Criteria

Scale (%)	Eligibility Criteria
85-100	Deserves the title of very good
65-84	Deserves a good title
45-64	Decent with the title sufficient
0-44	Not feasible

Based on the table above, if the developed product achieves results of more than or equal to 65%, the teaching material is declared suitable for use.

Results

Definition Stage

The definition stage in this research includes five steps of needs analysis in making teaching materials, namely as follows.

1. Front end analysis

Front-end analysis aims to surface and determine the basic problems faced in learning so that the development of teaching materials is needed. An overview of the facts in the field is obtained, and mathematical proof skills still need to be discussed in learning activities. This can be seen from the researchers' interviews with class XI IPA 2 students at SMAN 1 Sumberjaya.

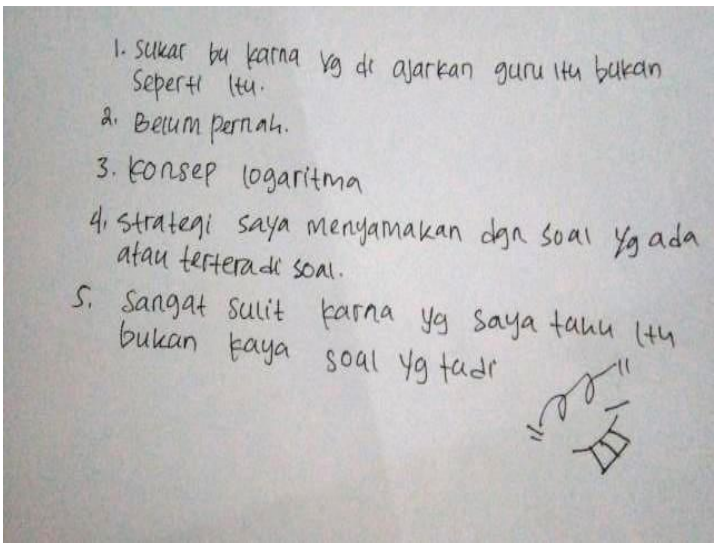


Figure 1. Interview Results

Based on the results of these interviews, researchers can conclude that most students who have studied logarithm material still need help when faced with questions regarding proof related to the basic material of logarithms themselves, namely the properties of logarithms.

2. Student analysis

Regarding knowledge, most students the researchers surveyed tended to be some of the best imitators. Whatever the teacher exemplifies, that is what is immediately imitated (Novitasari & Abduh, 2022). It is not uncommon for a student to even dare to reject the truth others convey just because the teacher does not say the same thing. High school students can recognise more complex cognitive domains based on cognitive development. Therefore, to support students' cognitive development, it is necessary to develop teaching materials that can also improve students' skills.

3. Concept analysis

The concept analysis made in developing this teaching material includes curriculum analysis, ability indicators, and learning objectives. This concept analysis can be summarised in Table 1.

Table 3. Concept Analysis

KI	KD	Learning Indicators	Learning objectives
Processing, reasoning and presenting in the concrete and abstract domains are related to the development of what they learn at school independently and can use methods according to scientific principles.	Read and present proofs relating to the properties of logarithms.	1) Read the proof of the statement proving the properties of logarithms. 2) Construct proof of the properties of logarithms.	1) Students can read the proof of the statement proving the properties of logarithms. 2) Students can construct a proof of the properties of logarithms.

4. Task analysis

Task analysis aims to identify the main skills that researchers will study. In this research, the main skill that will be studied is the ability to prove mathematics. Based on the results of testing the question instrument, two of the eight questions on the ability of mathematical proof did not meet the validity standards, so only six questions were taken as instruments in making valid materials.

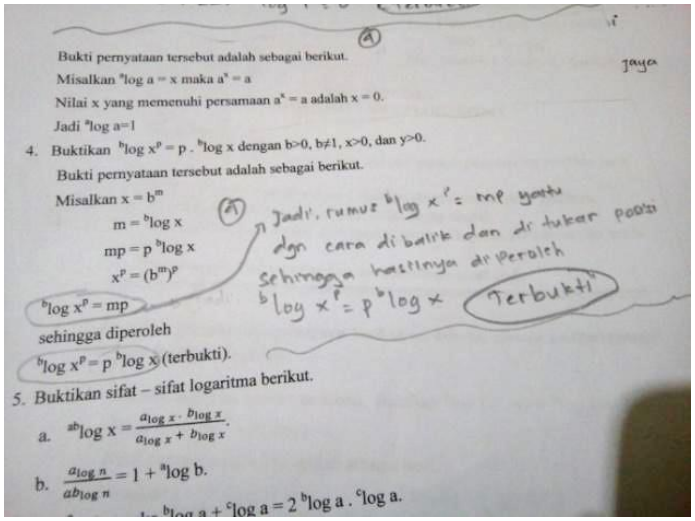

Bukti pernyataan tersebut adalah sebagai berikut.
Misalkan $^a\log a = x$ maka $a^x = a$
Nilai x yang memenuhi persamaan $a^x = a$ adalah $x = 0$.
Jadi $^a\log a = 1$
4. Buktikan $^b\log x^p = p \,.\, ^b\log x$ dengan $b>0, b\neq 1, x>0$, dan $y>0$.
Bukti pernyataan tersebut adalah sebagai berikut.
Misalkan $x = b^m$
$m = {}^b\log x$
$mp = p\,{}^b\log x$
$x^p = (b^m)^p$
$^b\log x^p = mp$
sehingga diperoleh
$^b\log x^p = p\,{}^b\log x$ (terbukti).
5. Buktikan sifat – sifat logaritma berikut.
a. $^{ab}\log x = \frac{a_{\log x} \cdot b_{\log x}}{a_{\log x} + b_{\log x}}$.
b. $\frac{a_{\log n}}{ab_{\log n}} = 1 + {}^a\log b$.
$^b\log a + {}^c\log a = 2\,{}^b\log a \,.\, {}^c\log a$.
Jaya
Jadi, rumus $^b\log x^p = mp$ yaitu dgn cara di balik dan di tukar posisi sehingga hasilnya diperoleh $^b\log x^p = p\,{}^b\log x$
Terbukti

Figure 2. Instrument trial results

From the test results, the student's way of answering questions could have been more systematic. It does not look nice, and it is very clear that students still need clarification about how to answer questions that, from their perspective, still need to be made familiar. This is because students need to get used to seeing questions in the form of proof. So, to better introduce students to the ability to prove, teaching materials that include examples of questions and guidance on the stages of working on proof questions are needed so that they can direct students in independent learning.

5. Formulation of learning objectives

This fifth step is a summary of the analysis results from the task analysis to determine the behaviour of the object. Based on the five steps at this definition stage, a concept analysis has been obtained to create a teaching material design according to the following results of the needs analysis.

Table 4. Formulation of learning objectives

Basic competencies	Mathematical Proof Ability Indicator	Question Indicator
Read and present proofs related to the properties of logarithms.	1) Ability to read mathematical proofs. 2) Ability to construct evidence	1) Determining truth or falsity by looking at the correspondence between the system of axioms, premises, and existing mathematical results (lemmas or theorems) with the flow of deductive reasoning through true or false statements. 2) compose a proof of a mathematical statement based on definitions, principles and theorems and write it in the form of a complete proof (direct or indirect proof)

This explanation appears that students experience difficulties where the knowledge that students have is limited to certain contexts. According to Hers (Stavrou, 2014, p. 1), the exact definition of evidence and its role varies based on the context and level of education. However, the general purpose of evidence is to verify, explain, communicate, and systematise reports in a deductive system.

Planning

The media chosen to realise the benchmark test created is to create a mathematics learning module on logarithm material, with a sub-chapter on the properties of logarithms.

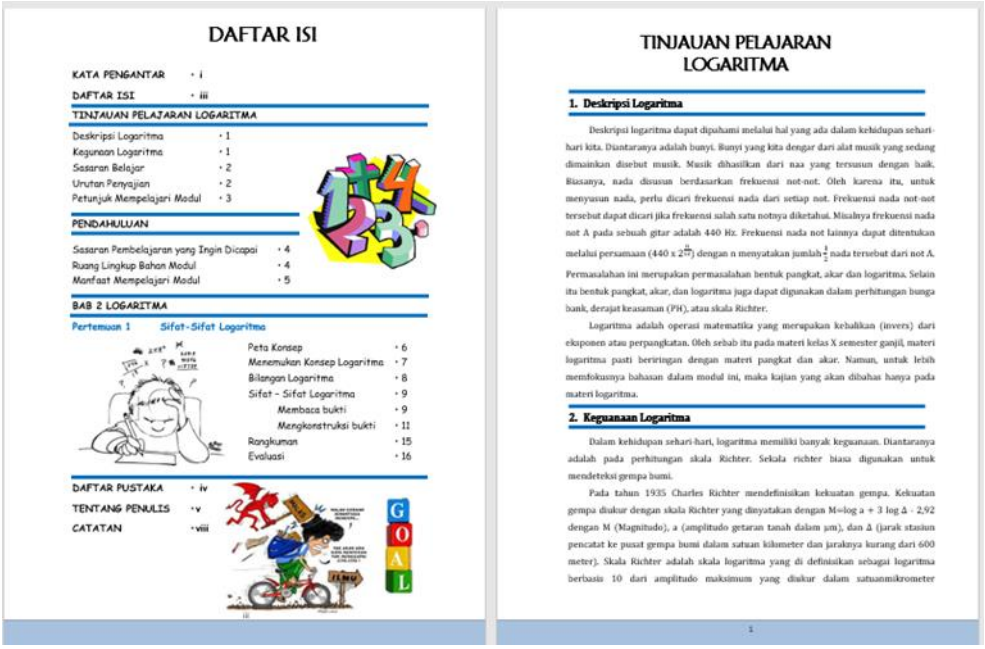


Figure 3. Media Selection

The format chosen in this research is the format for creating module teaching materials, adapted from LKPP and modified according to needs.

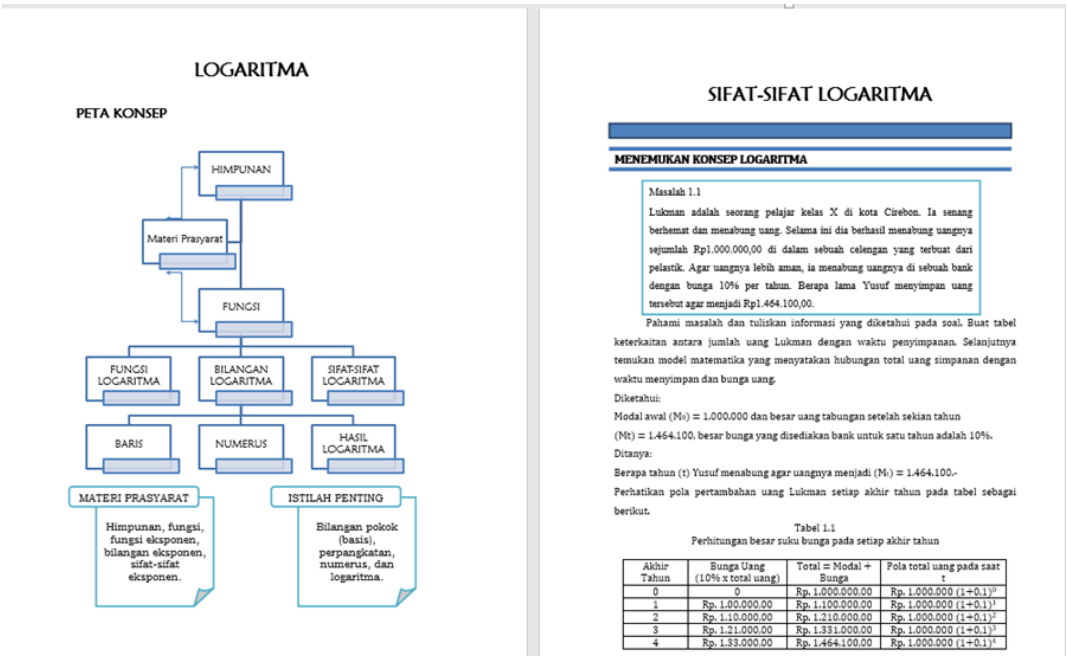


Figure 4. Concept Map and Discussion

The utilisation of idea maps in this particular scenario is quite commendable. Concept maps are highly efficient at visually deconstructing intricate concepts, facilitating readers' comprehension. I anticipate that the ensuing discussion will explore the concepts outlined in the concept map in greater detail, enhancing the reader's comprehension.

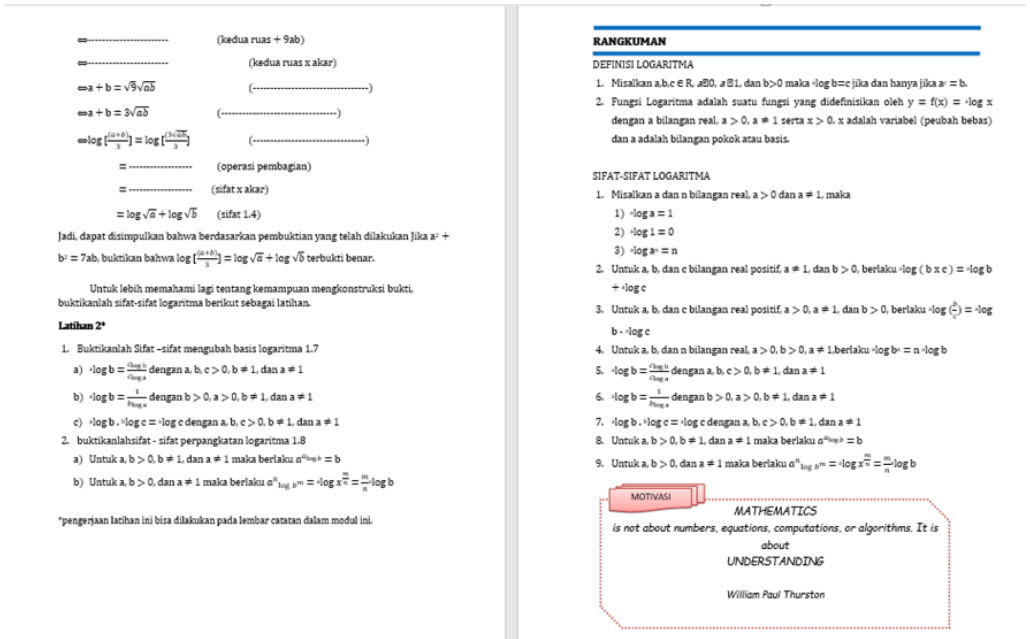
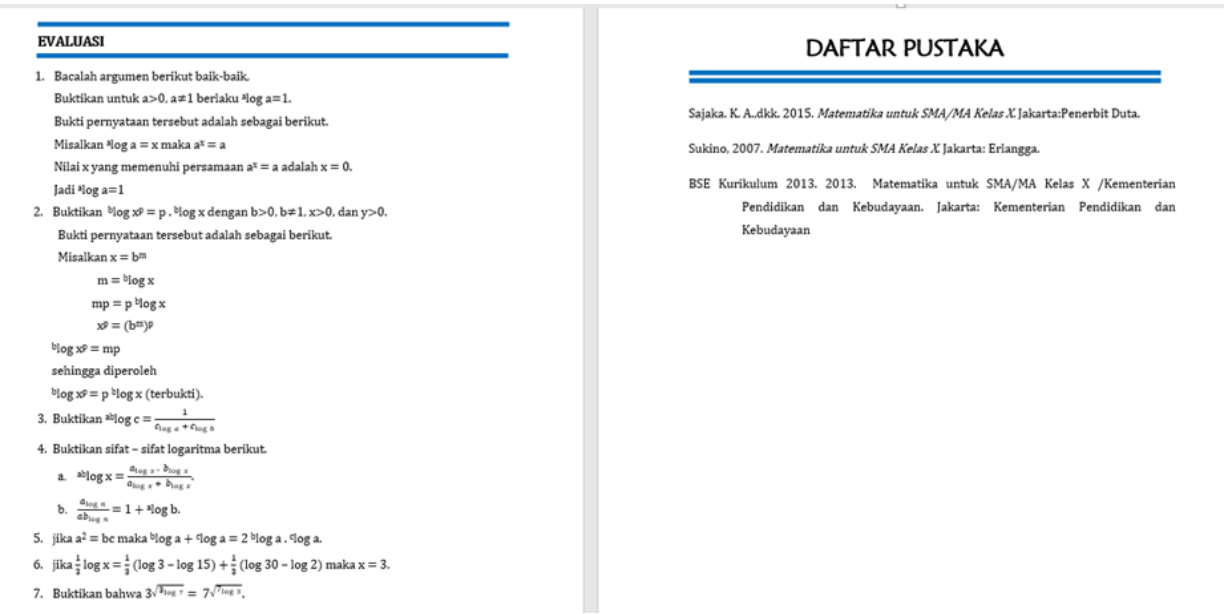


Figure 5. Summary and Motivation

The significance of including a summary and motive in a written work should not be undervalued. Figure 5 succinctly summarises the core content and catalyses readers to delve more into the subject

matter. It is crucial to ensure that readers comprehend the significance of the content discussed and are inspired to enhance their comprehension.



EVALUASI

1. Bacalah argumen berikut baik-baik.
   Buktikan untuk a>0, a≠1 berlaku $^{a}\log a=1$.
   Bukti pernyataan tersebut adalah sebagai berikut.
   Misalkan $^{a}\log a = x$ maka $a^x = a$
   Nilai x yang memenuhi persamaan $a^x = a$ adalah x = 0.
   Jadi $^{a}\log a=1$
2. Buktikan $^{b}\log x^p = p\,.\,^{b}\log x$ dengan b>0, b≠1, x>0, dan y>0.
   Bukti pernyataan tersebut adalah sebagai berikut.
   Misalkan $x = b^m$
   $m = {}^{b}\log x$
   $mp = p\,{}^{b}\log x$
   $x^p = (b^m)^p$
   $^{b}\log x^p = mp$
   sehingga diperoleh
   $^{b}\log x^p = p\,{}^{b}\log x$ (terbukti).
3. Buktikan $^{ab}\log c = \frac{1}{{}^{a}\log c + {}^{b}\log c}$
4. Buktikan sifat – sifat logaritma berikut.
   a. $^{ab}\log x = \frac{{}^{a}\log x \cdot {}^{b}\log x}{{}^{a}\log x + {}^{b}\log x}$.
   b. $\frac{{}^{a}\log n}{{}^{ab}\log n} = 1 + {}^{a}\log b$.
5. jika $a^2 = bc$ maka $^{b}\log a + {}^{c}\log a = 2\,{}^{b}\log a\,.\,{}^{c}\log a$.
6. jika $\frac{1}{3}\log x = \frac{1}{3}(\log 3 - \log 15) + \frac{1}{3}(\log 30 - \log 2)$ maka x = 3.
7. Buktikan bahwa $3^{\sqrt{{}^{3}\log 7}} = 7^{\sqrt{{}^{7}\log 3}}$.

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Figure 6. Evaluation questions and bibliography

The module is also equipped with guided practice questions, note sheets, author biographies and a discussion of evaluation questions. This module is designed to make learning easier for students. Before entering the main material, instructions for using the module and the learning objectives of the module are also given, enabling students to learn independently.

Development

The module that has been prepared is then validated by three experts. Teaching materials are validated using practicality sheets one teacher and two lecturers filled in. The expert validation results were 88.93% material experts and 85.08% media experts. This shows that the teaching materials are valid and can be used without revision. However, the lecturer provided some input. Several discussions, including suggestions and input provided by each validator, are formulated in the following table.

Table 5. Media Expert Validation Results

No	Material Expert Validator	Combined Validation Criteria	Validation Level
1	V1	85.08%	Very valid or usable without revision
2	V2		
3	V3		

Table 5 displays the validation outcomes obtained from media specialists about the supplied material. Through the use of specified validation criteria, the outcomes of this validation process offer an assessment of the material's validity and appropriateness without needing further adjustments. The validation results, as indicated in V1 (85.08%), demonstrate a high level of validity, suggesting that the

information is highly valid and does not require further modifications. Nevertheless, it is imperative to await validation outcomes from more experts, namely those mentioned in V2 and V3, to obtain a more comprehensive assessment of the material's overall validity.

Table 6. Validation Revision Results from Material Expert Validators

No	Validator	Suggestion	Revision
1	V1	Improve the layout system for writing the table of contents, author biodata, and subtitles.	Improving the systematic order of the table of contents, tidying up the structure of writing biodata, and writing subtitles.
2	V2	1. Pay more attention to the contrast of the colour of the writing and the background so that it looks comfortable to look at 2. Some parts of the sentences are spaced incorrectly	1. Change the background and colour combination of the module design 2. Double-check the wording and use of spaces

Table 6 displays the outcomes of validation modifications conducted by material specialists on the supplied material. The results encompass recommendations for enhancements provided by each validator to enhance the quality of the material. For V1, the proposed enhancements involve refining the layout framework for composing the table of contents, author biodata, and subtitles. Regarding V2, recommendations for enhancement include prioritising the colour contrast between the text and background to ensure a visually pleasing experience. Additionally, reviewing and rectifying any unsuitable language and spacing issues is advisable. By integrating recommendations for enhancements provided by these subject matter specialists, it is anticipated that the information can be enhanced to achieve superior readability and overall visual representation.

Table 7. Material Expert Validation Results

No	Material Expert Validator	Combined Validation Criteria	Validation Level
1	V1	88.93%	Very valid or usable without revision
2	V2		
3	V3		

The validation results from material specialists on the evaluated material are presented in Table 7. Through the use of specified validation criteria, the outcomes of this validation process offer an assessment of the material's validity and appropriateness without necessitating further adjustments. V1's validation findings in the first row indicate a high level of validity at 88.93%, suggesting that the material is suitable for usage without any more adjustments. Nevertheless, to guarantee the overall excellence of the material, it remains imperative to await validation outcomes from additional experts, as indicated in V2 and V3. This approach will lead to a more thorough comprehension of the material's validity and appropriateness for use.

Table 8. Validation Revision Results from Material Expert Validators

No	Validator	Suggestion	Revision
1	V1	1. Lack of systematicity in delivering material 2. The module is not only dominated by blue	1. Organize the systematic delivery of material 2. Added several colours to the cover and contents
2	V2	1. There is even more variety in the example questions; make sure there are ones that are related to applications in everyday life 2. Improve the introduction and add material related to the properties of logarithms, which are related to everyday life	1. Additional sample questions include a guide to working on the questions 2. Improvements to the foreword and addition of material related to applications in life.

After validation, it is declared valid. Next is to carry out a simulation. This simulation was carried out on high school students in class. Next, results were obtained at the limited trial stage, as in the following table.

Table 9. Analysis of Student Response Results

No	Respondent	Response Criteria	Combined Response Criteria	Interpretation
1	R1	82.14%	82.5%	Very strong
2	R2	82.14%		
3	R3	87.5%		
4	R4	82.14%		
5	R5	80.35%		
6	R6	82.14%		
7	R7	82.14%		
8	R8	82.14%		
9	R9	80.35%		
10	R10	83.92%		

Analysis of the results of student responses shows that the interpretation is appropriate with a good predicate. This reflects that after being given the module on the properties of logarithms based on mathematical proof abilities, they better understand them, which can help students learn mathematical proof skills. However, until now, most students still need to improve mathematical proof skills (Nurrahmah & Karim, 2018; . However, based on the research that has been done, this module on the properties of logarithms based on mathematical proof capabilities has been declared valid and can be used.

CONCLUSION

Based on the results of the analysis that has been carried out, it can be concluded that designing a logarithm teaching material based on mathematical proof skills is carried out in 4 stages of design development, which have been recommended by Thiagarajan. This research is limited to the third stage: definition, planning, and development.

The results of the analysis that have been carried out show that errors often occur, one of which is in reading evidence from the properties of logarithms and constructing evidence related to the properties of logarithms. Apart from that, the anticipation given by the teacher during learning and presented in the module can overcome students' difficulties regarding mathematical proof skills in logarithm material.

This teaching material was validated by three experts: two mathematics education lecturers and one mathematics subject teacher. Validation of teaching materials is carried out using practicality sheets. The expert validation results were 88.93% of material experts and 85.08% of media experts. This shows that the teaching materials are valid and can be used without revision. The results of the combined practicality sheet filled in by students showed 82.5%; this is included in the very strong/very practical category.

The results of this research are supported by several previous studies related to mathematical proof-based teaching materials. These include the use of teaching materials on the ability to write evidence (Kusumawati & Kurniawan, 2020), Nur'afni et al. (2019) regarding proof-based binomial teaching materials, and the application of student worksheet design for learning proof systems (Huwaa & Matitaputty, 2019) produces valid, practical, and effective teaching materials and student worksheets.

The suggestions for further research are expected to be able to develop teaching materials with other materials, and the stages are carried out to implement the revised teaching materials in schools so that the design of the teaching materials created is optimal.

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Author Information

Ida Hamidah
https://orcid.org/0000-0003-3953-7510
STAI Al-Bahjah
Cirebon
Indonesia
Contact e-mail: *idahamidah@staialbahjah.ac.id*

Susilawati
https://orcid.org/0000-0001-6780-7907
STAI Al-Bahjah
Cirebon
Indonesia
Contact e-mail: *susilawati@staialbahjah.ac.id*

Shafira Meiria Rahmasari
https://orcid.org/ 0000-0002-8189-1049
STAI Al-Bahjah
Cirebon
Indonesia
Contact e-mail: *shafirameiria@staialbahjah.ac.id*

Abdul Khalim
https://orcid.org/0009-0005-9104-8628
Beijing Jiaotong University
Beijing
China
Contact e-mail: 20129005@bjtu.edu.cn