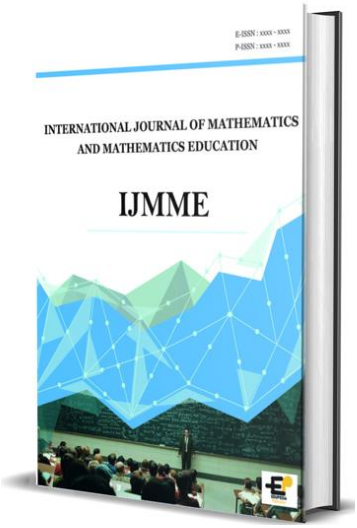


https://journals.eduped.org/index.php/IJMME

Mathematical Modeling of Forgetfulness and Memorization of Mathematical Concepts

Ebenezer Bonyah

Department of Mathematics Education
Akenten Appiah-Menka University of Skills Training and Entrepreneurial Development, Ghana

Ernest Larbi

Department of Mathematics Education
Akenten Appiah-Menka University of Skills Training and Entrepreneurial Development, Ghana

Raphael Owusu

Department of Mathematics Education
Akenten Appiah-Menka University of Skills Training and Entrepreneurial Development, Ghana

To cite this article:

Boyah, *et. al.,* (2023). Mathematical Modeling of Forgetfulness and Memorization of Mathematical Concepts. *International Journal of Mathematics and Mathematics Education (IJMME), 1*(1), 31-50
https://doi.org/10.56855/ijmme.v1i1.212

IJMME International Journal of Mathematics and Mathematics Education

February 2023, Vol. 01, No. 01, 31-50 doi: 10.56855/ijmme.v1i1.212

Mathematical Modeling of Forgetfulness and Memorization of Mathematical Concepts

Ebenezer Bonyah, Ernest Larbi, Raphael Owusu

Article Info

Article History

Received:
21 December 2022
Accepted:
27 Januari 2023

Keywords

Sumudu transform
Forgetfulness
Memorisation
Mittag-Leffler function
Stability

Abstract

Memorization and forgetfulness in Mathematics Education have been major issues in most developing countries, including Ghana. This work explored forgetfulness and memorization in the context of the fractional calculus domain using the Mittag-Leffler function. The Sumudu transform was employed to obtain a special solution. The existence and uniqueness of solutions was established using the concept of the fixed-point theory. It is established that fractional order plays a major role in memorization and forgetfulness of mathematical concepts. It is concluded that factors that enhance the teaching and learning of mathematical concepts should be encouraged.

Introduction

As mathematics educators strive to improve teaching to enhance learners' learning output, one major concern, or worry, has been the issue of forgetfulness that results from the loss of learned materials from memory. Forgetfulness is inevitable in human recalling ability. Most learners experience a loss of learned materials after a course is completed (Kamuche & Ledman, 2005). However, people are educated, particularly in mathematics, to enable them to use the knowledge gained to solve problems encountered in their daily lives. Therefore there is a need to retain the knowledge gained so that it can be applied to solve daily problems when necessary. As knowledge of mathematics continues to be part of human lives, the problem is that learners forget the content they have learned due to several experiences (Ramirez, McDonough, & Jin, 2017). According to the authors, what is even more disturbing is when learners forget what they have learned soon after class. Forgetfulness is a situation where learners experience the loss of what they have learned. This occurs despite the efforts made by students to study (Ramirez, G., 2017). Whilst learning can be controlled, we have little or no control over the process of forgetting (Simon & Bulko, 2014). Therefore, researchers and educators try to discover what causes forgetfulness and find ways to help learners remember what they have learned.

The issue of forgetfulness is attributed to factors such as interference, disuse of information, and absence of cues for retrieval of information (Ramirez et al. 2017). Forgetfulness comes about when learned materials are not revisited after some time, entry knowledge structures are weak, and there is an inability to gain a deep understanding of the learned materials (Deslauriers & Wieman, 2011; Conway et al. 1991). Knowledge of such factors from research informs educators' preparation and delivery of instruction. Such factors enable educators to organise teaching and learning in ways that ensure that knowledge is developed on a solid foundation of existing knowledge. Building on existing knowledge improves the understanding of learned concepts and leads to knowledge retention in long-term memory. Bruner (1966) asserts that new knowledge that has no connection to prior knowledge is easily forgotten. Educators further organise the learning environment so that it is conducive to learning and attracts the active involvement of all learners. The National Council of Teachers of Mathematics (1989) states that we understand mathematics through doing and thinking about what we do. This supports the Chinese proverb that states:

I hear, I forget; I see, I remember; I do, I understand.

Indeed, understanding is a key ingredient for the effective retention of learned materials and hence it promotes good recall ability. Learning with understanding enables one to retrieve information from memory with little effort.

The two major theoretical factors underlying forgetfulness are interference and decay/disuse. Interference is a situation in which one has difficulty recalling what has been learned due to an obstruction by previously learned material. Interference can be proactive inhibition or retroactive inhibition (Sprinthall & Sprinthall, 1990). Proactive inhibition occurs when what was learned earlier disrupts the recall of newly learned material. In other words, the existing memories disrupt or interfere with the retention of new mathematical concepts. For instance, a person may have learned earlier that the circumference of a circle is $2\pi r$. If it is learned later that the area of a circle is πr^2 and the earlier learned formula for the circumference of a circle inhibits or obstructs the learning of the formula for an area of a circle, this is known as proactive inhibition.

In this case, efforts to retrieve the formula for the area of a circle are hampered by an earlier learned formula for the circumference of a circle. The second type of interference is retroactive inhibition, in which the learning of new concepts interferes with the retention of previously learned concepts. Similar to the earlier example, this is the case where the learning of the new concept of area of a circle prevents the recall of the formula for the circumference of a circle learned earlier (Sprinthall & Sprinthall, 1990). Interferences may occur when a person considers two learned materials to be similar.

The other factor underlying forgetfulness is decay or disuse. Decay occurs when the material learned has not been revisited for some time. In other words, decay results from the “passive loss of the memory trace” that comes from a lack of rehearsal or use (Sprinthall & Sprinthall, 1990, p. 293). Decay happens when there has not been frequent encounters with the information. When information is not used frequently, it becomes weak, due to its dormant status, and fades from memory. It is worthy to note that the first encounter with new material is stored in the short-term memory, which gets lost in a few minutes if left unrehearsed. Memory enhancement thus necessitates frequent visits to learned materials in order for them to be pushed into long-term memory for easy recall (Sprinthall & Sprinthall, 1990). Forgetfulness of learned mathematical concepts is a worrying and disturbing issue in mathematics teaching and learning that must be addressed. It has attracted the attention of many researchers. For instance, Moore (2002) asserts that an unimaginable aspect of memory is being able to recall details of novels, even when you do not intend to, but finding it difficult to remember learned concepts for passing an examination. This calls for the attention of researchers and educators to determine the rate of forgetfulness among learners and recommend what needs to be done, as educators, to aid memory recall.

Mathematics educators seem to have focused most of their research activities on improving mathematics instruction and on ways children learn mathematics. The qualitative and quantitative research approaches have been the key drivers in this regard. The modeling aspect of quantitative research in mathematics education has received minimal attention. Mathematical modeling has been recognised as an indispensable tool for providing qualitative information about phenomena in the absence of data. Mathematical modeling, in the context of fractional calculus, has some merit because it predicts by taking into account memory effect and present condition. Some of the operators, such as Caputo- Fabrizio and Atangana-Baleanu, have crossover properties that help in predicting outcomes accurately. This paper therefore uses Atangana-Baleanu in the Caputo sense, to present some useful qualitative information about the dynamics of forgetfulness and memorization of mathematical concepts, together with the uniqueness and existence of solutions to the forgetfulness and memorization models

Material and Method

The study followed a quantitative research approach and used a descriptive survey design. According to Best and Khan (2006), a descriptive survey is a type of research that is used to describe a behavior, or a characteristic of a population, without any manipulation. Thus, it is used to describe the current state of affairs with no concern for what caused the behaviour. It mostly deals with what presently exists (Ary et al., 2010, Best and Khan, 2006). The study sought to analyse and describe the students' ability to model the concept of forgetfulness based on what they have been taught and their ability to use the knowledge gained to model the behaviour under study. In other words, the concern was to determine how the students could model the rate at which decay occurs in human learning and retention due to interference.

Participants

Twenty post graduate students, in the second semester of their study programme, participated in the study. They were tasked to formulate a Mathematical Model based on Forgetfulness and Memorization of General Mathematics Concepts using a fractional calculus perspective. The students had taken several content courses, both as undergraduates and in the first semester of their graduate studies. Some of the courses were Algebra, Calculus, Probability and Statistics, Differential Equations, and Partial Differential Equations. These courses serve as the foundational knowledge students needed for the modeling. At the time of the study, the graduate students had also been introduced to fractional calculus. Based on the course or modules they had taken, it was believed that they were knowledgable enough to do the modeling.

Data Collection

Data were collected from students who agreed to participate in the study after its rationale was explained to them. The study was motivated by the desire to determine the amount of knowledge the students could recall to perform the modeling. The goal was to determine how new information interfered with previously learned concepts and the application of mathematical concepts required to complete the modeling. Data were collected based on the participants' ability to recall and perform the modeling as required. Thus, the data obtained from the participants described their retention of the knowledge involved in the modeling.

Results

Model Formulation

This section is devoted to the mathematical formation of memorization and forgetfulness of mathematical concepts. The full meaning of mathematical concept (A), a person loses some mathematical concept at a rate and becomes a high memory prone person (N). A person loses mathematical concepts at a rate and becomes a low memory of those concepts (L). The rate at which a high concept person gets back to full mathematical concepts is denoted by γ_1, and the rate at which a low concept person attains full mathematical concepts is denoted by γ_2. The natural decay, or loss of a mathematical concept, is denoted by μ. The recruitment rate into full memorization of mathematical concept formation is Λ. The Fgure 1 represents the various interactions against the various compartments.

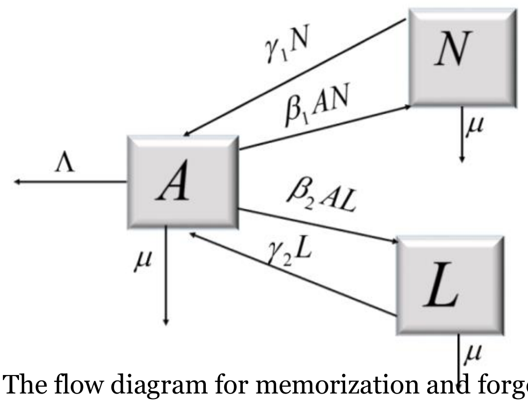


Figure 1: The flow diagram for memorization and forgetfulness

$$\begin{aligned} \frac{dA}{dt} &= \Lambda - \beta_1 AN - \beta_2 AL + \gamma_1 N + \gamma_2 L - \mu A, \\ \frac{dN}{dt} &= \beta_1 AN - (\gamma_1 + \mu) N, \\ \frac{dL}{dt} &= \beta_2 AL - (\gamma_2 + \mu) L. \end{aligned} \tag{1}$$

The model is reformulated in the context of the Mittag-Leffler function (Atangana-Baleanu operator) which retains the post and present data for prediction. The operator has a crossover property that ensures accurate prediction:

$$\begin{aligned} {}_0^{AB}D_t^{\sigma} A(t) &= \Lambda - \beta_1 A(t) N(t) - \beta_2 A(t) N(t) + \gamma_1 N(t) + \gamma_2 L(t) - \mu A(t) \\ {}_0^{AB}D_t^{\sigma} N(t) &= \beta_1 A(t) N(t) - (\gamma_1 + \mu) N(t), \\ {}_0^{AB}D_t^{\sigma} L(t) &= \beta_2 A(t) L(t) - (\gamma_2 + \mu) L(t). \end{aligned} \tag{2}$$

Stability Analysis

In this aspect of the work, we shall examine the stability of the model.

Lemma 1. *The closed set* $\phi = \left\{ (A, H, L) \in R_+^3 : A + H + L \le \dfrac{\Lambda}{\mu} \right\}$ is considered positively invariant with regard to the model (2).

Proof. For model (2), the fractional derivative (FD) of the entire population is formed by

$${}_0^{AB}D_t^{\sigma} N = \Lambda - \mu N(t) \le \Lambda - \mu N(t), \tag{3}$$

Utilizing the concepts of Laplace transform, the equation (3) has the following,

$$N(t) \le \left(\frac{B(\sigma)}{B(\sigma) + (1-\sigma)\mu} N(0) + \frac{(1-\sigma)\Lambda}{Q(\sigma) + (1-\sigma)\mu} \right) E_{\sigma,1}\left(-\beta t^{\sigma}\right) + \frac{\sigma(\Lambda)}{Q(\sigma) + (1-\sigma)\mu} E_{\sigma,\sigma+1}\left(-\beta t^{\sigma}\right) \tag{4}$$

Where $\beta = \dfrac{\sigma\mu}{Q(\sigma) + (1-\sigma)\mu}$ and $E_{\sigma,\beta}$ constitute the two parameter Miltcy-Leftler function (ML).

$$E_{\sigma_1,\sigma_2}(\varepsilon) - \sum_{k}^{w} \frac{\varepsilon^{-q}}{T(\sigma_2 - \sigma_1 k)} + O\left(|\varepsilon|^{-1-w}\right)\left(|\varepsilon| \to \infty, \frac{\sigma, \Pi}{2} < |\arg(\varepsilon)| \le \Pi\right) \tag{5}$$

It is easy to observe that $N(t) \leq \frac{\Lambda}{\mu}$ as $t \to \infty$. Thus, the total solution of the model (2) with the associated initial conditions belonging to ϕ for every $t > 0$. Therefore, ϕ is deemed as a positively invariant region with regard to model (2).

The system model (2) possesses three steady state points: E_0, E_1, and E_2. However, they only meets the requirements of the system model (2}) with $A \geq 0,\ N \geq,\ L \geq 0$. The other two equilibria do not have any physical interpretation and therefore, are not considered. For the purpose of system model (2), we concentrate on the Equilibrium $E_0 = \left(\frac{\Lambda}{\mu}, 0, 0\right)$. Clearly, E_0 depicts the forgetfulness and memorization states.

The Jacobian matrix evaluated at is given by:

$$J_{E_0} = \begin{pmatrix} -\mu & \frac{-\beta\Lambda}{\mu} + \gamma_1 & \frac{-\beta_2\Lambda}{\mu} + \gamma_2 \\ 0 & \frac{\beta_1\Lambda}{\mu} - (\gamma_1 + \mu) & 0 \\ 0 & 0 & \frac{\beta_2\Lambda}{\mu} - (\gamma_2 + \mu) \end{pmatrix}$$

Considering theorem (2) of Bonyah (2020), the basic threshold number based on system model (2) is expressed as:

$$R_0 = \frac{\Lambda\beta_1}{\mu(\mu + \gamma_1)} + \frac{\Lambda\beta_2}{\mu(\mu + \gamma_2)}$$

Remark 1. *The memorization and forgetfulness free* E_0 *hinged on* E_0 *is denoted asymptotically stable wherever* $R_0 < 1$ (Bonyah, 2020).

Special solution for the model via Mittag-Leffler function

The aim of this section is to present a special solution for the model (2) via the Sumudu transformation with iterative technique. The following are obtained:

$$\begin{aligned} &\frac{Q(\sigma)}{1-q}\left(qT(q+1)\right)E_q\left(-\frac{p^q}{1-q}\right)\left(S\left(A(t)\right) - A(0)\right) = \\ &\qquad S\left[\Lambda - \beta_1 A(t)N(t) - \beta_2 A(t)L(t) + \gamma_1 N(t) + \gamma_2 L(t) - \mu A(t)\right], \\ &\frac{Q(\sigma)}{1-q}\left(qT(q+1)\right)E_q\left(-\frac{p^q}{1-q}\right)\left(S\left(N(t)\right) - N(0)\right) = S\left[\beta_1 A(t)N(t) - (\gamma_1 + \mu)N(t)\right], \\ &\frac{Q(\sigma)}{1-q}\left(qT(q+1)\right)E_q\left(-\frac{p^q}{1-q}\right)\left(S\left(L(t)\right) - L(0)\right) = S\left[\beta_2 A(t)L(t) - (\gamma_2 + \mu)L(t)\right] \end{aligned} \tag{6}$$

Reorganizing system (6) we have:

Making use of inverse Sumudu transform with respect to equation (7), we have the following:

$$\begin{aligned}
S\left(A(t)\right) &= A(0) + \frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)} \times \\
&\quad S\left[\Lambda - \beta_1 A(t)N(t) - \beta_2 A(t)L(t) + \gamma_1 N(t) + \gamma_2 L(t) - \mu A(t)\right], \\
S\left(N(t)\right) &= N(0) + \frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)} S\left[\beta_1 A(t)N(t) - (\gamma_1+\mu)N(t)\right], \\
S\left(L(t)\right) &= L(0) + \frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)} S\left[\beta_2 A(t)L(t) - (\gamma_2+\mu)L(t)\right]
\end{aligned} \tag{7}$$

The following recursive formula from system equation (8) can be obtained:

$$\begin{aligned}
A(t) &= A(0) + S^{-1}\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)} \times \\
&\quad S\left[\Lambda - \beta_1 A(t)N(t) - \beta_2 A(t)L(t) - \gamma_1 N(t) + \gamma_2 L(t) - \mu A(t)\right], \\
N(t) &= N(0) + S^{-1}\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)} S\left[\beta_1 A(t)N(t) - (\gamma_1+\mu)N(t)\right], \\
L(t) &= L(0) + S^{-1}\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)} S\left[\beta_2 A(t)L(t) - (\gamma_2+\mu)L(t)\right].
\end{aligned} \tag{8}$$

The following recursive formula from system equation (8) can be obtained:

The solution of the system of equation (9) is presented as:

$$\begin{aligned}
A(t) &= \lim_{n\to\infty} A_{(n)}(t), \\
N(t) &= \lim_{n\to\infty} N_{(n)}(t), \\
L(t) &= \lim_{n\to\infty} l_{(n)}(t).
\end{aligned}$$

Stability Analysis and Iterative Solution

Assuming that a Banach Space $\left(Z,\|\cdot\|\right)$ and R^* be a self-map of Z. Supposing that a particular recursive procedure of the form $y_{n+1}=g\left(R^*,y_n\right)$. Let $F\left(R^*\right)$ be a fixed point set of R^* that is non-empty and y_m converges to a point $R^*\in F\left(R^*\right)$. Let $\left\{Z_n^*\subseteq Z\right\}$ and define $\left\|z_{n+1}^*-g\left(R^*,z_n^*\right)\right\|=e_n$. The iterative method $y_{n+1}=g\left(R^*,y_n\right)$ is said to be R^* stable if $\lim_{m\to\infty}e^m=0$ implies $\lim_{n\to\infty}z_n^*=r^*$. In the same way, we assume that the z^*n sequence has an upper limit; else, the sequence does not converge. If all of these conditions are satisfied for $y_{n+1}=R^*$ and m, considered as Picard's iteration as described in (Wang, Yang, Ma, & Sun, 2014). Consequently, the iteration is $R*$ stable. We will then state the theorem below:

Theorem 1. *Let $\left(Z,\|\cdot\|\right)$ be a Banach space, and let R^* be a Z satisfying self-map,*

$$\left\|R_z^*-R_y^*\right\|\le C\left\|z-R_z^*\right\|+c\left\|z-y\right\|,$$

for every $z,y\in Z$ where $0\le C,0\le c<1$. Supposing that R^* is Picard R^* stable. Taking into consideration the recursive formula as

$$A_{n+1}(t)=A_n(t)+S^{-1}\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)}\times$$

$$S\left[\Lambda-\beta_1A_n(t)N_n(t)-\beta_2A_n(t)L_n(t)-\gamma_1N_n(t)+\gamma_2L_n(t)-\mu A_n(t)\right],$$

$$N_{n+1}(t)=N_n(t)+$$

$$S^{-1}\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)}S\left[\beta_1A_n(t)N_n(t)-(\gamma_1+\mu)N_n(t)\right],$$

$$L_{n+1}(t)=L_n(t)+$$

$$S^{-1}\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)}S\left[\beta_2A_n(t)L_n(t)-(\gamma_2+\mu)L_n(t)\right].$$

Where $\dfrac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\dfrac{p^q}{1-q}\right)}$ *is the fractional Langrange Multiplier*

Theorem 2. *Let G be a self-map defined as*

$$G\left(A_n(t)\right)=A_{n+1}(t)=A_n(t)+S^{-1}\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)}\times$$

$$S\left[\Lambda-\beta_1A_n(t)N_n(t)-\beta_2A_n(t)L_n(t)-\gamma_1N_n(t)+\gamma_2L_n(t)-\mu A(t)\right],$$

$$G\left(N_n(t)\right)=N_{n+1}(t)=N_n(t)+$$

$$S^{-1}\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)}S\left[\beta_1A_n(t)N_n(t)-(\gamma_1+\mu)N_n(t)\right],$$

$$G\left(L_n(t)\right)=L_{n+1}(t)=L_n(t)+$$

$$S^{-1}\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)}S\left[\beta_2A_n(t)L_n(t)-(\gamma_2+\mu)L_n(t)\right].$$

It is G-Stable in $L^1(a,b)$ *if*

$$\begin{cases}1-\gamma_1+\gamma_2-uf(w)-\beta_1(M+L)g(\gamma)-\beta_2(P+H)h(\gamma)<1,\\ 1-(\gamma_1+u)f_1(w)+\beta_1(M+L)g_1(\gamma)<1,\\ 1-(\gamma_2+u)f_2(w)+\beta_2(P+H)h_1(\gamma)<1,\end{cases}\tag{10}$$

The initial step indicates that G has a fixed point. In finalizing this, we find the followings for all $(m,n)\in N\times N$:

$$G\left(A_n(t)\right)-G\left(A_m(t)\right)=$$

$$S^{-1}\left[\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)}S\left[\Lambda-\beta_1A_n(t)N_n(t)-\beta_2A_n(t)L_n(t)-\gamma_1N_n(t)+\gamma_2L_n(t)-\mu A_n(t)\right]\right],$$

$$A_n(t)-A_m(t)-$$

$$S^{-1}\left[\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)}S\left[\Lambda-\beta_1A_m(t)N_m(t)-\beta_2A_m(t)L_m(t)-\gamma_1N_m(t)+\gamma_2L_m(t)-\mu A_m(t)\right]\right],$$

$$G\left(N_n(t)\right)-G\left(N_m(t)\right)=S^{-1}\left[\frac{(1-q)}{Q(\varpi)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)}S\left[\beta_1A_n(t)N_n(t)-(\gamma_1+\mu)N_n(t)\right]\right],$$

$$N_n(t)-N_m(t)-S^{-1}\left[\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)}S\left[\beta_1A_m(t)N_m(t)-(\gamma_1+\mu)N_m(t)\right]\right],$$

$$G(L_n(t)) - G(L_m(t)) = S^{-1}\left[\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)} S\left[\beta_2 A_n(t) L_n(t) - (\gamma_2+\mu) L_n(t)\right]\right].$$

$$L_n(t) - L_m(t) - S^{-1}\left[\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)} S\left[\beta_2 A_m(t) L_m(t) - (\gamma_2+\mu) L_m(t)\right]\right]. \tag{11}$$

Taking into consideration, the first equation of (11) and taking the norm,

$$\begin{aligned} &\left\| G(A_n(t)) - G(A_m(t)) \right\| = \\ &\left\| S^{-1}\left[\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)} \times \right.\right. \\ &\left. S\left[\Lambda - \beta_1 A_n(t) N_n(t) - \beta_2 A_n(t) L_n(t) - \gamma_1 N_n(t) + \gamma_2 L_n(t) - \mu A_n(t)\right]\right], \qquad (12) \\ &A_n(t) - A_m(t) - S^{-1}\left[\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)} \times \right. \\ &\left.\left. S\left[\Lambda - \beta_1 A_m(t) N_m(t) - \beta_2 A_m(t) L_m(t) - \gamma_1 N_m(t) + \gamma_2 L_m(t) - \mu A_m(t)\right]\right]\right\| \end{aligned}$$

Utilizing the triangular inequality, we obtain the following

$$\begin{aligned} &\left\| G(A_n(t)) - G(A_m(t)) \right\| = \\ &\left\| S^{-1}\left[\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)} \times \right.\right. \\ &\left. S\left[\Lambda - \beta_1 A_n(t) N_n(t) - \beta_2 A_n(t) L_n(t) - \gamma_1 N_n(t) + \gamma_2 L_n(t) - \mu A_n(t)\right]\right], \qquad (13) \\ &A_n(t) - A_m(t) - S^{-1}\left[\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)} \times \right. \\ &\left.\left. S\left[\Lambda - \beta_1 A_m(t) N_m(t) - \beta_2 A_m(t) L_m(t) - \gamma_1 N_m(t) + \gamma_2 L_m(t) - \mu A_m(t)\right]\right]\right\| \end{aligned}$$

Simplifying, equation (13) turns to

$$\left\|G\left(A_n(t)\right)-G\left(A_m(t)\right)\right\| \leq \left\|A_n(t)-A_m(t)\right\|+S^{-1}\times$$
$$\left[\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)}S\left(\left\|-\beta_1 A_n(t)\left(N_n-N_m\right)\right\|+\left\|-\beta_2 L_m(t)\left(A_n-A_m\right)\right\|+\left\|-\mu\left(A_n-A_m\right)\right\|\right)\right], \tag{14}$$

Let assume that

$$\left\|N_{(n)}(t)-N_{(m)}(t)\right\| \cong \left\|A_{(n)}(t)-A_{(m)}(t)\right\|,$$
$$\left\|L_{(n)}(t)-L_{(m)}(t)\right\| \cong \left\|A_{(n)}(t)-A_{(m)}(t)\right\|,$$

The equation (14) implies that

$$\left\|G\left(A_n(t)\right)-G\left(A_m(t)\right)\right\| \leq \left\|A_n(t)-A_m(t)\right\|+S^{-1}\times\left[\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)}\times\right. \tag{15}$$
$$\left.S\left(\left\|-\beta_1 A_n\left(A_n-A_m\right)\right\|+\left\|-\beta_2 A_m\left(A_n-A_m\right)\right\|+\left\|-\mu\left(A_n-A_m\right)\right\|\right)\right],$$

Since S_n, S_m depicts convergent sequence therefore are bounded, therefore, there exist M and L different positive constant so as

$$\left\|A_m\right\|<L,\left\|A_n\right\|<M,(m,n)\in N\times N. \tag{16}$$

Equations (15) and (16) imply

$$\left\|G\left(A_{(n)}(t)\right)-G\left(A_{(m)}\right)\right\| \leq$$
$$\left\|A_{(n)}-A_{(m)}\right\|\left[1+\gamma_1+\gamma_2-\mu f(\omega)-\beta_1(M+L)g(\gamma)-\beta_2(P+H)h(\gamma)\right] \tag{17}$$

the functions *f* and *g* are the consequences of $S^{-1}\left[\left(\frac{(1-q)}{Q(\sigma)qT(q+1)E_q\left(-\frac{p^q}{1-q}\right)}\right)S\right].$

Similarly,

$$\left\|G\left(N_{(n)}(t)\right)-G\left(N_{(m)}\right)\right\| \leq \left[1-\left(\gamma_1+u\right)f_1(w)+\beta_1(M+L)g_1(\gamma)\right]\left\|N_{(n)}-N_{(m)}\right\|, \tag{18}$$ (18)

$$\left\|G\left(L_{(n)}(t)\right)-G\left(L_{(m)}\right)\right\| \leq \left[1-\left(\gamma_2+u\right)f_2(w)+\beta_2(P+H)h_1(\gamma)\right]\left\|L_{(n)}-L_{(m)}\right\|, \quad (19)$$

Where,

$$\begin{cases} 1-\gamma_1+\gamma_2-uf(w)-\beta_1(M+L)g(\gamma)-\beta_2(P+H)h(\gamma)<1, \\ 1-\left(\gamma_1+u\right)f_1(w)+\beta_1(M+L)g_1(\gamma)<1, \\ 1-\left(\gamma_2+u\right)f_2(w)+\beta_2(P+H)h_1(\gamma)<1, \end{cases}$$

This shows that the non-linear G-self mapping has a fixed point. Next, we show that G satisfies all the conditions of Theorem 2. Let (5), (17), and (19) hold; therefore, using

$$c=(0,0,0), C=\begin{cases} \{1-\gamma_1+\gamma_2-uf(w)-\beta_1(M+L)g(\gamma)-\beta_2(P+H)h(\gamma)\}, \\ \{1-\left(\gamma_1+u\right)f_1(w)+\beta_1(M+L)g_1(\gamma)\}, \\ \{1-\left(\gamma_2+u\right)f_2(w)+\beta_2(P+H)h_1(\gamma)\}. \end{cases}$$

The above expression shows that for the non-linear mapping G, all conditions of Theorem 1 exist and satisfied. Hence, G is Picard G-stable.

Uniqueness of Special Solution

In this section, we made use of the iteration form to demonstrate the uniqueness of equation (2). We will first assume a solution for Equation (2) by which, for a large number w, the unique solution converges. We take the following Hillbert Space $F=K\left((a,b)\times(0,T)\right)$, Which can be defined as follows: $x:(a,b)\times(0,T)\to R$, such that $\iint uvdudv<\infty$

$$D(A,N,L)=\begin{cases} \Lambda-\beta_1 AN-\beta_2 AL+\gamma_1 N+\gamma_2 L-\mu A, \\ \beta_1 AN-\left(\gamma_1+\mu\right)N, \\ \beta_2 AL-\left(\gamma_2+\mu\right)L. \end{cases}$$

Thus,

The inner product of

5. $$D\left(\left(M_{11}-M_{12}, M_{21}-M_{22}, M_{31}-M_{32}\right),\left(q_1,q_2,q_3\right)\right),$$

where $\left(M_{11}-M_{12}\right),\left(M_{21}-M_{22}\right)$ and $\left(M_{31}-M_{32}\right)$ are solutions to the system.

Notwithstanding,

$$D\left(\left(M_{11}-M_{12},M_{21}-M_{22},M_{31}-M_{32}\right),\left(q_1,q_2,q_3\right)\right)$$

$$=\begin{cases}(-\beta_1\left(M_{11}-M_{12}\right)\left(M_{21}-M_{22}\right)-\beta_2\left(M_{11}-M_{12}\right)\left(M_{31}-M_{32}\right)+\\ \gamma_1\left(M_{21}-M_{22}\right)+\gamma_2\left(M_{31}-M_{32}\right)-\mu\left(M_{11}-M_{12}\right),q_1),\\ \left(\beta_1\left(M_{11}-M_{12}\right)\left(M_{21}-M_{22}\right)-\left(\gamma_1+\mu\right)\left(M_{21}-M_{22}\right),q_2\right)\\ \left(\beta_2\left(M_{11}-M_{12}\right)\left(M_{31}-M_{32}\right)-\left(\gamma_2+\mu\right)\left(M_{31}-M_{32}\right),q_3\right).\end{cases} \quad (20)$$

Examining the first equation,

$$\begin{aligned}&(-\beta_1\left(M_{11}-M_{12}\right)\left(M_{21}-M_{22}\right)-\beta_2\left(M_{11}-M_{12}\right)\left(M_{31}-M_{32}\right)+\gamma_1\left(M_{21}-M_{22}\right)+\gamma_2\left(M_{31}-M_{32}\right)-\mu\left(M_{11}-M_{12}\right),q_1)\cong\\ &\quad -\left(\beta_1\left(M_{11}-M_{12}\right)\left(M_{21}-M_{22}\right),q_1\right)+\left(-\beta_2\left(M_{11}-M_{12}\right)\left(M_{31}-M_{32}\right),q_1\right)+\\ &\quad \left(\gamma_1\left(M_{21}-M_{22}\right),q_1\right)+\left(\gamma_2\left(M_{31}-M_{32}\right),q_1\right)+\left(-\mu\left(M_{11}-M_{12}\right),q_1\right)\end{aligned} \quad (21)$$

Supposing, $\left(M_{11}-M_{12}\right)\cong\left(M_{21}-M_{22}\right)\cong\left(M_{31}-M_{32}\right).$

Equation (21) gives

$$\left(-\beta_1\left(M_{11}-M_{12}\right)^2-\beta_2\left(M_{11}-M_{12}\right)^2+\gamma_1\left(M_{21}-M_{22}\right)+\gamma_2\left(M_{31}-M_{32}\right)-\mu\left(M_{11}-M_{12}\right),q_1\right)$$

which means

$$\begin{aligned}&\left(-\beta_1\left(M_{11}-M_{12}\right)^2-\beta_2\left(M_{11}-M_{12}\right)^2+\gamma_1\left(M_{21}-M_{22}\right)+\gamma_2\left(M_{31}-M_{32}\right)-\mu\left(M_{11}-M_{12}\right),q_1\right)\\ &\cong-\left(\beta_1\left(M_{11}-M_{12}\right)^2,q_1\right)+\left(-\beta_2\left(M_{11}-M_{12}\right)^2,q_1\right)+\left(\gamma_1\left(M_{11}-M_{12}\right),q_1\right)+\left(\gamma_2\left(M_{11}-M_{12}\right),q_1\right)+\left(-\mu\left(M_{11}-M_{12}\right),q_1\right)\\ &\le\beta_1\left\|\left(M_{11}-M_{12}\right)^2\right\|\left\|q_1\right\|+\beta_2\left\|\left(M_{11}-M_{12}\right)^2\right\|\left\|q_1\right\|+\gamma_1\left\|\left(M_{11}-M_{12}\right)\right\|\left\|q_1\right\|+\gamma_2\left\|\left(M_{11}-M_{12}\right)\right\|\left\|q_1\right\|+\mu\left\|\left(M_{11}-M_{12}\right)\right\|\left\|q_1\right\|\\ &\qquad=\left(\beta_1\omega_1+\beta_2\omega_1+\gamma_1+\gamma_2+\mu\right)\left\|\left(M_{11}-M_{12}\right)\right\|\left\|q_1\right\|\end{aligned} \quad (22)$$

Following the same pattern, we can get the following from the second equation,

$$\begin{aligned}&\left(\beta_1\left(M_{11}-M_{12}\right)\left(M_{21}-M_{22}\right)-\left(\gamma_1+\mu\right)\left(M_{21}-M_{22}\right),q_2\right)\\ &\le\beta_1\left\|\left(M_{21}-M_{22}\right)^2\right\|\left\|q_2\right\|+\left(\gamma_1+\mu\right)\left\|\left(M_{21}-M_{22}\right)\right\|\left\|q_2\right\|\\ &\qquad=\left(\beta_1\omega_2+\left(\gamma_1+\mu\right)\right)\left\|\left(M_{21}-M_{22}\right)\right\|\left\|q_2\right\|\end{aligned} \quad (23)$$

In the same way the third equation of system (20) yields

$$\begin{aligned}&\left(\beta_2\left(M_{11}-M_{12}\right)\left(M_{31}-M_{32}\right)-\left(\gamma_2+\mu\right)\left(M_{31}-M_{32}\right),q_3\right)\le\\ &\left(\beta_2\omega_3+\left(\gamma_2+\mu\right)\right)\left\|\left(M_{31}-M_{32}\right)\right\|\left\|q_3\right\|\end{aligned} \quad (23)$$

Substituting equation (21 – 24) into equation (20) we obtain

$$D\left(\left(M_{11}-M_{12},M_{21}-M_{22},M_{31}-M_{32}\right)\times,\left(q_1,q_2,q_3\right)\right)\leq \begin{cases}\left(\beta_1\omega_1+\beta_2\omega_1+\gamma_1+\gamma_2+\mu\right)\left\|\left(M_{11}-M_{12}\right)\right\|\left\|q_1\right\|,\\ \left(\beta_1\omega_2+\left(\gamma_1+\mu\right)\right)\left\|\left(M_{21}-M_{22}\right)\right\|\left\|q_2\right\|,\\ \left(\beta_2\omega_3+\left(\gamma_2+\mu\right)\right)\left\|\left(M_{31}-M_{32}\right)\right\|\left\|q_3\right\|.\end{cases} \tag{25}$$

Nevertheless, for adequately large values of w_i with $i=1,2,3,$ the approximate solution tends to the analytical solution. Making use of the topological concepts five, very small positive parameters exits, i.e., l_{w_1}, l_{w_2} and l_{w_3} such that

$$\left\|A-M_{11}\right\|,\left\|A-M_{12}\right\|<\frac{l_{w_1}}{3\left(\left(\beta_1\omega_1+\beta_2\omega_1+\gamma_1+\gamma_2+\mu\right)\left\|\left(M_{11}-M_{12}\right)\right\|\right)}$$

$$\left\|N-M_{21}\right\|,\left\|N-M_{22}\right\|<\frac{l_{w_2}}{3\left(\left(\beta_1\omega_2+\left(\gamma_1+\mu\right)\right)\left\|\left(M_{21}-M_{22}\right)\right\|\left\|q_2\right\|\right)}$$

$$\left\|L-M_{31}\right\|,\left\|L-M_{32}\right\|<\frac{l_{w_2}}{3\left(\left(\beta_2\omega_3+\left(\gamma_2+\mu\right)\right)\left\|\left(M_{31}-M_{32}\right)\right\|\left\|q_3\right\|\right)}$$

Linking the solution to the right side of equation (25) and applying the triangle inequality and taking $W=\max\left(w_1,w_2,w_3\right)$, $l=\max\left(l_{w_1},l_{w_2},l_{w_3}\right)$. We get

$$\begin{cases}\left(\beta_1\omega_1+\beta_2\omega_1+\gamma_1+\gamma_2+\mu\right)\left\|\left(M_{11}-M_{12}\right)\right\|\left\|q_1\right\|,\\ \left(\beta_1\omega_2+\left(\gamma_1+\mu\right)\right)\left\|\left(M_{21}-M_{22}\right)\right\|\left\|q_2\right\|,\\ \left(\beta_2\omega_3+\left(\gamma_2+\mu\right)\right)\left\|\left(M_{31}-M_{32}\right)\right\|\left\|q_3\right\|.\end{cases}<\begin{cases}l,\\ l,\\ l.\end{cases}$$

As we take into consideration l to be negligible, therefore, one has a topological idea as

$$\begin{cases}\left(\beta_1\omega_1+\beta_2\omega_1+\gamma_1+\gamma_2+\mu\right)\left\|\left(M_{11}-M_{12}\right)\right\|\left\|q_1\right\|,\\ \left(\beta_1\omega_2+\left(\gamma_1+\mu\right)\right)\left\|\left(M_{21}-M_{22}\right)\right\|\left\|q_2\right\|,\\ \left(\beta_2\omega_3+\left(\gamma_2+\mu\right)\right)\left\|\left(M_{31}-M_{32}\right)\right\|\left\|q_3\right\|.\end{cases}<\begin{cases}0,\\ 0,\\ 0.\end{cases}$$

Nevertheless, it is obvious that

$$\left(\beta_1\omega_1+\beta_2\omega_1+\gamma_1+\gamma_2+\mu\right)\left\|\left(M_{11}-M_{12}\right)\right\|\left\|q_1\right\|\neq 0,\qquad \left(\beta_1\omega_2+\left(\gamma_1+\mu\right)\right)\left\|\left(M_{21}-M_{22}\right)\right\|\left\|q_2\right\|\neq 0,$$

$$\left(\beta_2\omega_3+\left(\gamma_2+\mu\right)\right)\left\|\left(M_{31}-M_{32}\right)\right\|\left\|q_3\right\|\neq 0$$

Hence, we have

$$\left\|M_{11}-M_{12}\right\|=0,\left\|M_{21}-M_{22}\right\|=0,\left\|M_{31}-M_{32}\right\|=0.$$

Which yields that

$$M_{11} - M_{12} = 0, M_{21} - M_{22} = 0, M_{31} - M_{32} = 0$$

This concludes the proof of uniqueness.

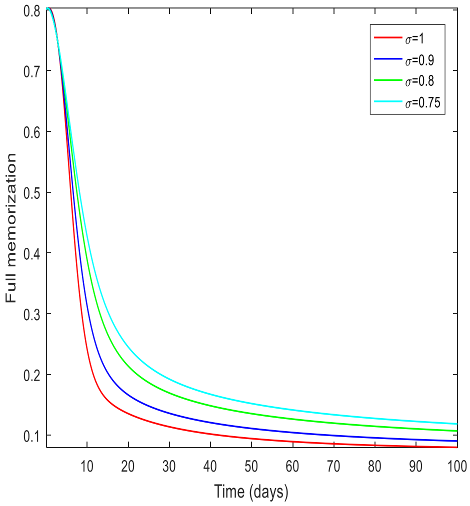
Discussion

Numerical results and discussions

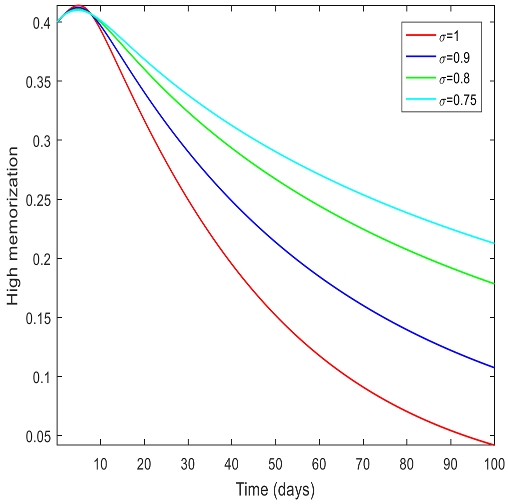
For the purpose of illustration, in this section, a numerical simulation results based on the Adams–Moulton method as in Gomez-Aguilar (2020) for forgetfulness and memorization, model (1) is investigated. The following parameter values were utilized: $\Lambda = 0.08$, $\beta_1 = 0.05$, $\beta_2 = 0.4$, $\gamma_1 = 0.02$, $\gamma_2 = 0.02$, μ=0.01

Figure 2 shows the numerical simulation results of the parameter values stated above. Figure 2(a) depicts a full memorization of general mathematics concepts which reduces as more students either get to a high memorization state or less memorization of mathematical general concepts. It can be observed that, as the fractional order derivative increases, the level of memorization decreases. This is supported by many mathematics educational learning theories including Piaget and Bruner. According to Piaget, the law of decay would set in as the students are not revising frequently the general mathematical concepts taught.

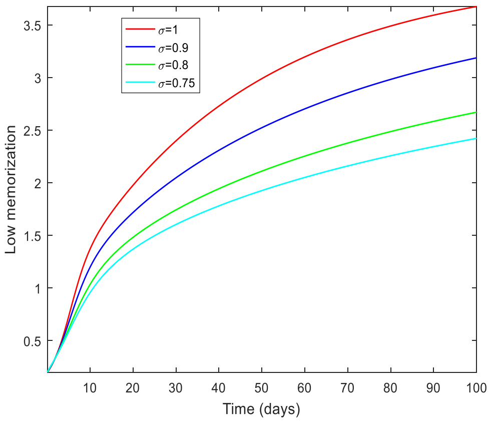
Figure 2(b) depicts the high-level memorization of mathematical concepts and as the fractional order derivative increases towards 1, the level of memorization is reduced. This is typically associated with learners as they believe that they know a particular concept and therefore it is not necessary to spend more time revising the already stored schemas. The intelligent quotient (IQ) students, according to constructivists such as Piaget and Bruner, suggest practices to avoid decay if the already learned concepts are not used. Figure 2(c) is the Low memorization of general mathematics concepts and as the fractional order derivative increases from 0.75 to one (1), the level of forgetfulness also rises. Many mathematics educational learning theories support the fact that teaching and learning should be organized in an activity-based manner where learners take an active part. In Ghana, for example, the new basic mathematics school curricula have spelt out the teachers' role purely as a facilitator and learners are expected to lead the learning processes.



(a)



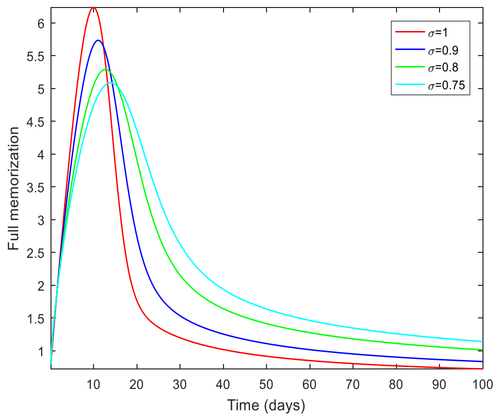
(b)



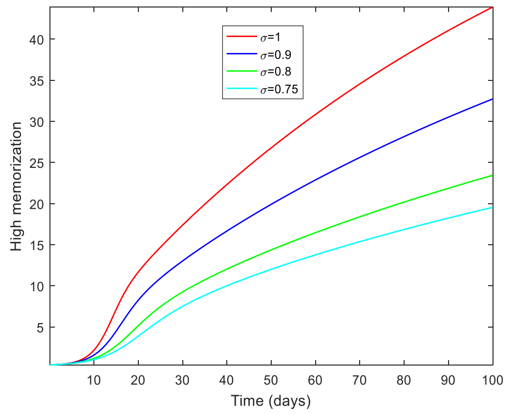
(c)

Figure 2: Numerical simulation for forgetfulness and memorization model (2) via Mittag- Leffler generalized function at σ=1, 0.9, 0.80, 0.75

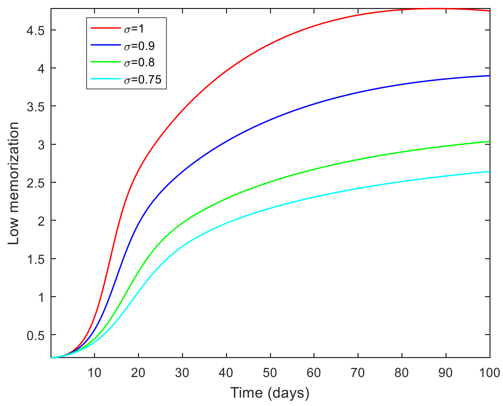
Figure 3, $\beta_1 = 0.5$ was varied in order to examine the dynamics associated with equation model (2). Figure 3(a) is the full memorization of mathematical concepts and individuals reduce their mathematical concepts as the fractional order derivative increases towards 1. The constructivist point of view about learning of concepts depicts that frequent practicing is important. In Figure 3(b), the high-level memorization of general mathematical concepts moves up as the fractional order derivative increases. This also may imply in some of the learning theories that one can have high memorization of general mathematics concepts through creating a good learning environment to enhance retention of the concept studied. Mathematics teachers may have to increase the effort that leads to practice of mathematical general concepts. Figure 3(c) indicates that as fractional order increases, the level of low memorization also goes up. This may be due to a poor learning environment for general mathematics learning concept formation. Efforts from all players in education must be utilized to achieve this goal. Teachers therefore must be provided with all the necessary teaching and learning materials, as well as frequent workshops, to keep them on track.



(a



(b



(c

Figure 3: Numerical simulation for forgetfulness and memorization model (2) via Mittag- Leffler generalized function at σ=1, 0.9, 0.80, 0.75 when $\beta_1 = 0.5$.

In Figure 4, $\beta_2 = 0.7$ was varied in order to investigate the dynamics of the equation model (2). The Figure 4(a) indicates that as the fractional order derivative increases, the full memorization of mathematical concepts decreases. Piaget believes that as concepts are not used, the law of decay sets in and learners forget what they have been taught. In Figure 4(b), the high-level memorization of mathematical concepts goes as fractional order increases and this indicates that good learning environments and teachers' work output are of high standard. Many learning theories, including Skinner as a behaviourist theory, emphasise that learners must be presented with environments that cause certain behaviours to occur and discourage undesirable behaviours in the formation of mathematical concepts. This also implies that one can have high memorization of general mathematics concepts, but if an effort is not made to improve it in the long term, the law of decay may set in. Figure 4(c) indicates that as fractional order increases, the level of low memorization also goes down. This may be due to poor reinforcement, as indicated by Skinner, and also to a poor learning environment which would lead to weak general mathematics concept formation. Efforts from all players in the educational sector must be utilised to achieve this goal. Therefore teachers must be provided with all the necessary teaching and learning materials, as well as frequent workshops, to keep them on track.

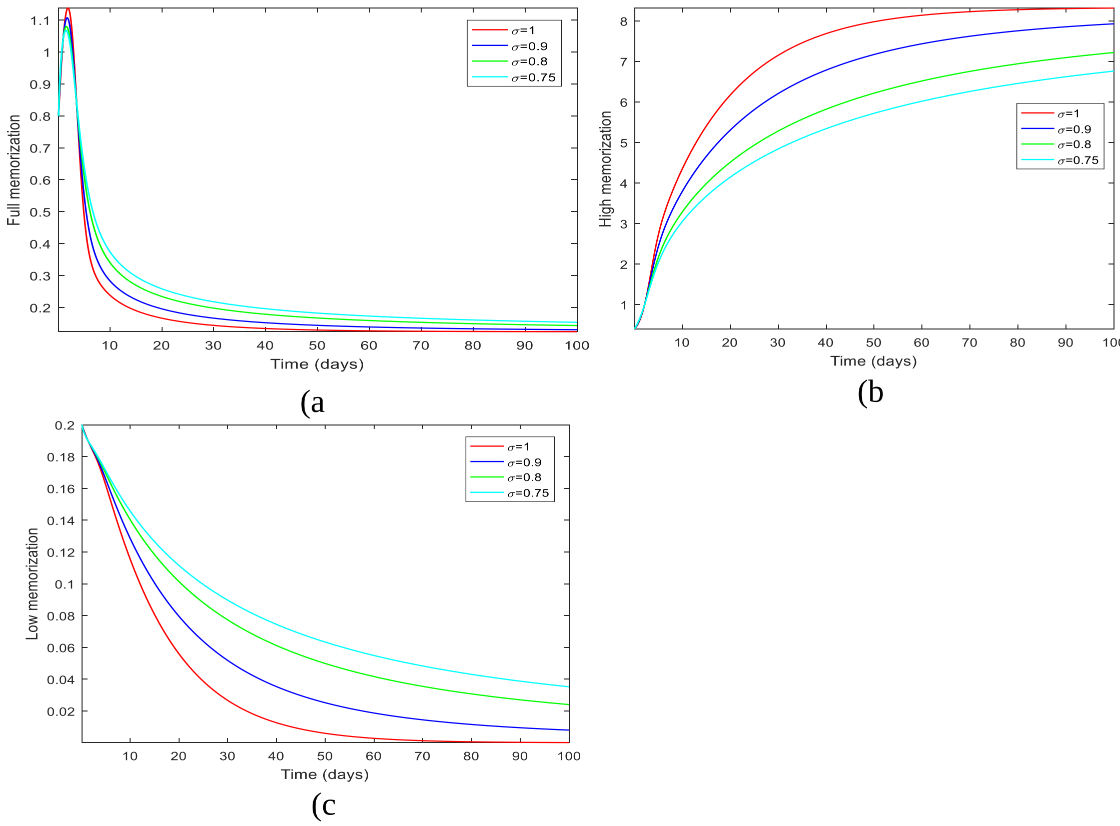


Figure 4: Numerical simulation for forgetfulness and memorization model (2) via Mittag- Leffler generalized function at σ=1, 0.9, 0.80, 0.75 when $\beta_2 = 0.7$

Conclusion

In this paper, a mathematical model on memorization and forgetfulness of mathematical concepts was formulated. The study hinged on a fractional derivative in the concept Mittag-Leffler function, which is a non-singular kernel and non-local. The fractional order has an effect on the dynamics of the model. The recall of mathematical concepts, from the numerical analysis, depicted that decay would set in if learners were not practicing what they had been taught. It was also inferred from the numerical results that factors that bring about forgetfulness should not be encouraged. In the future, it is suggested that other aspects of mathematics education can be modelled using fractional calculus in order to obtain some useful qualitative information.

Conflict of Interests: The authors declare that there is no conflict of interest regarding the publication of this paper.

References

Bonyah, E. (2020). Fractional conformable and fractal-fractional power-law modeling of coronavirus. *Mathematics in Engineering, Science & Aerospace (MESA), 11*(3), 577-594.

Bruner, J. S. (1966). *Toward a theory of instruction.* Cambridge, Massachusetts:: Belknap Press.

Conway, M. A., Cohen, G., & Stanhope, N. (1991). On the very long-term retention of knowledge acquired through formal education: Twelve years of cognitive psychology. *Journal of Experimental Psychology: General, 120*(4), 395 - 409.

Deslauriers, L., & Wieman, C. (2011). Learning and retention of quantum concepts with different teaching methods. *Physical Review Special Topics-Physics Education Research, 7*(1), 010101. doi:10.1103/PhysRevSTPER.7.010101

Gomez-Aguilar, J. (2020). Chaos and multiple attractors in a fractal{fractional shinriki's oscillator model. *Physica A: Statistical Mechanics and its Applications, 539*, 122918. doi:https://doi.org/10.1016/j.physa.2019.122918

Kamuche, F. U., & Ledman, R. E. (2005). Relationship of time and learning retention. *Journal of College Teaching & Learning (TLC), 2*(8). doi:10.19030/tlc.v2i8.1851

Moore, G. (2002). What do you mean, you've forgotten: An investigation into mathematics and memory. Retrieved from https://www.m-a.org.uk/resources/Vol-31-No5

National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Retrieved from http://www.nctm.org/

Ramirez, G. (2017). Motivated forgetting in early mathematics: A proof-of-concept study. *Frontiers in Psychology, 8*, 2087. doi:10.3389/fpsyg.2017.02087

Ramirez, G., McDonough, I. M., & Jin, L. (2017). Classroom stress promotes motivated forgetting of mathematics knowledge. *Journal of Educational Psychology, 109*(6), 812-825. doi:10.1037/edu0000170

Šimon, J., & Bulko, M. (2014). A simple mathematical model of cyclic circadian learning. *Journal of Applied Mathematics,, 2014.*

Sprinthall, N. A., & Sprinthall, R. C. (1990). *Educational psychology: A developmental approach 5th edition.* New York, N.Y: McGraw-Hill, Inc.

Wang, Z., Yang, D., Ma, T., & Sun, N. (2014). Stability analysis for nonlinear fractional-order systems based on comparison principle. *Nonlinear Dynamics,, 75*(1), 387-402.

Author Information

Ebenezer Bonyah
https://orcid.org/0000-0001-8763-6164
Department of Mathematics Education, Faculty of Applied Science and Mathematics Eduation Akenten Appiah-Menka University of Skills Training and Entrepreneurial Development, Kumasi, Ghana.
Contact e-mail: *ebbonyah@gmail.com*

Ernest Larbi
https://orcid.org/0000-0003-3652-5223
Department of Mathematics Education, Faculty of Applied Science and Mathematics Eduation Akenten Appiah-Menka University of Skills Training and Entrepreneurial Development, Kumasi, Ghana.
Contact e-mail: *ertlarbi@yahoo.com*

Raphael Owusu
https://orcid.org/0000-0003-4072-0013
Department of Mathematics Education, Faculty of Applied Science and Mathematics Eduation Akenten Appiah-Menka University of Skills Training and Entrepreneurial Development, Kumasi, Ghana.
Contact e-mail: *rapowusu1994@gmail.com*