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The Impact of Inductive Teaching on Pre-Tertiary Students' Academic Performance in Solving Circle Theorem Problems

Thomas Gona Akwasi Dimaweh[1*] , Ebenezer Bonyah[2], Benjamin Adu Obeng[3]

[1,2,3]Akenten Appiah-Menka University of Skills Training and Entrepreneurial Development, Ghana

*Corresponding author: thomasdimaweh18@gmail.com

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ABSTRACT

Purpose – This study was motivated by the persistent challenges students face in solving geometry problems, particularly circle theorems. It aimed to investigate how an inductive teaching approach affects pre-tertiary students' performance in solving circle theorem problems.

Methodology – A mixed-methods approach grounded in the pragmatist paradigm was used, adopting a quasi-experimental pre-test–post-test control-group design. Using purposive and stratified sampling, 84 second-year students were selected and assigned to experimental and control groups. The experimental group was taught through the inductive approach, while the control group received conventional instruction. Data were collected through achievement tests and interviews, and analysed using descriptive statistics, t-tests, and thematic analysis.

Findings – Students demonstrated significant conceptual, procedural, and factual difficulties in solving circle-theorem problems. However, the experimental group performed significantly better in the post-test than the control group. Interview findings also indicated that students perceived the inductive approach as more engaging, interactive, and effective in improving understanding. The study concluded that the inductive intervention positively influenced both achievement and attitudes toward learning the circle theorem.

Novelty – The study's novelty lies in examining the effect of inductive teaching on students' performance in solving circle-theorem problems within a pre-tertiary education context.

Significance – The study contributes to improving mathematics teaching strategies by emphasizing the potential of inductive teaching to enhance students' achievement and attitudes toward complex concepts such as circle theorems, thereby supporting wider implementation and future research.

Keywords: Academic performance; Circle theorem; Inductive teaching; Pre-tertiary students'.

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1. Introduction

Mathematics is highly significant for the advancement and technological development of the world (Gula & Jojo, 2024). It is because of this that curriculum planners emphasize the importance of mathematics to the growth of every nation, which, as a result, has made it compulsory in Ghana for every student to learn mathematics from the start of the childhood stage. Mathematical concepts and principles are utilised in many fields, including economics, engineering, medicine, business, and science (Maskar & Herman, 2023). According to Ameen et al. (2022), the community views mathematics as the foundation of knowledge in science and technology, which is critical to a country's social and economic advancement.

As reported by Hissan and Ntow (2021) , mathematics is crucial in numerous areas of human activity. For instance, Kashefi et al. (2013) contend that mathematics serves as the basis and a key component of most applied engineering courses. According to Imoko and Isa (2015), mathematics should be prioritised from an early age in a country's educational system because it is the foundation for cognitive growth and innovative thinking, as cited by Ameen et al. (2022). Despite the importance of education and, in particular, mathematics, numerous studies have reported a decline in the performance of pre-tertiary students, as few students pursue mathematics-related disciplines, with the majority shying away from it (e.g., Adolphus, 2011; Oladosu, 2014; Laborde, 2005). Ameen et al. (2022) highlighted that students' performance in mathematics has been consistently disappointing in both internal and external exams (Akanmu et al., 2014; Salman, 2017). Additionally, the researchers noted that the WAEC (2019) Chief Examiners' reports revealed inconsistent competence in mathematics by students in the WASSCE. Among the main areas as outlined in the mathematics syllabus (CRDD, 2010) is Geometry (Badu-Domfeh, 2020). It is quite disheartening that, among the areas of mathematics in which students struggle and, as a result, strive to even get a pass in exams, Geometry is one of them (Adolphus, 2011). Among the aspects of mathematics that are taught from the elementary level in school to higher education is Geometry. Understanding shapes and their properties helps us see mathematics as part of our everyday experiences.

According to Serin (2018), geometry is a mathematical discipline that studies shapes and spatial relationships. Pierce and Stacey (2011) emphasize that this area of mathematics is essential for fostering students' problem-solving and innovative thinking skills. Thus, it is evident that comprehending geometric concepts is crucial for representing and solving problems in mathematics and other disciplines (Herbst et al., 2005). Additionally, geometry is an essential requirement for science courses, including chemistry and physics. Ozerem (2012) states that "studying geometry is a crucial aspect of learning mathematics, as it enables students to analyze and interpret the world around them and provides them with tools applicable to other mathematical areas." This implies that a person's comprehension of their environment, as well as their ability to apply this knowledge across various mathematical fields, depends on their understanding of geometry (Aidoo-Bervell, 2021).

Educators, therefore, need a thorough understanding of geometry and how to effectively teach it to students. One aspect of mathematics, seen as a fundamental subject that students find challenging to understand, is geometry (Oladosu, 2014). It is widely recognised that achievement levels in geometry are generally low. Consequently, mathematics and geometry often become a source of anxiety and fear for many students (Akın & Cancan, 2007). According to Mullis et al. (2016), Ghanaian basic school students performed poorly in mathematics, and the report further highlights that a significant number of students, specifically those from Ghana, who participated encountered considerable challenges as they strived to comprehend concepts and relationships in geometry.

As a result, Lappan (2000) also documented that American high school students performed poorly on all TIMSS geometry tasks. Notably, one major aspect of geometry that students perform poorly in as a result of the difficulties and challenges they face in learning and understanding is Circle theorems. Circle theorems are fundamental principles in geometry that describe the relationships and properties of circles and the lines and angles associated with them. Particularly in the study of circle geometry, these theorems are essential for comprehending and applying geometric principles. A thorough grasp of circle theorems is essential for students, as it helps develop their analytical and problem-solving skills. The study of circle theorems is integral to the broader field of geometry. Circle theorems learning not only provides essential tools for solving geometric problems but also fosters logical reasoning and critical thinking.

Multiple WAEC reports (e.g., 2018) have consistently highlighted the ongoing challenges students face with circle geometry. These reports indicate that students often struggle to interpret and apply deductive reasoning to identify pertinent geometric properties. This deficiency also affects their ability to utilize theoretical statements deductively. Similarly, Suglo et al. (2023) found that many students incorrectly answered question 3a, which concerned circle theorems, indicating limited knowledge and understanding of the topic (WAEC, 2020). Furthermore, part (b) of question 11 received poor responses from most candidates, underscoring students' difficulties and lack of enthusiasm for circle theorems.

Several factors contribute to these challenges, impacting students' overall performance in geometry. One primary factor is the perceived abstract nature of circle geometry (Segbefia, 2020). Students often see these concepts as theoretical and disconnected from practical applications, making it difficult to grasp and retain the material (Bosson-Amedenu, 2018). This perception of abstraction leads to a lack of engagement and interest, further hindering understanding. Additionally, the complexity of the deductive reasoning required to solve circle-theorem problems poses a significant barrier. Students must apply logical steps to prove theorems and solve related questions, which can be difficult without a solid foundation in deductive reasoning and critical thinking skills (Prahmana & D'Ambrosio, 2020). Many students struggle to identify and use relevant geometric properties and principles, leading to errors and incomplete solutions in examinations (WAEC, 2011; WAEC, 2012).

Instructional methods and teacher preparedness also play crucial roles in student performance (Prince & Felder, 2006; WAEC, 2019). Inadequate teaching strategies that fail to make the subject engaging and accessible can exacerbate students' difficulties. The conventional approach to teaching circle theorems in Ghana, which relies heavily on lectures and oral explanations, tends to focus more on the teacher than the student. This method often results in students who are proficient in performing calculations but find it challenging to apply mathematical concepts and skills to practical, real-world situations (Aidoo-Bervell, 2021). Constructing, visualizing, and justifying geometric concepts, such as circle theorems, is a skill that many students at the pre-tertiary level lack (Aidoo-Bervell, 2021). Teachers may not always provide sufficient explanations, examples, or connections to real-world contexts, hindering students' ability to understand and apply circle theorems (Ntow & Hissan, 2021). Alio and Habor-Peter (2000) similarly link inadequate teaching strategies to subpar exam results in mathematics.

1.1. Statement of the Problem

Recent studies reported by Badu-Domfeh (2020) indicate that geometry is a fundamental component of mathematics that has attracted significant interest from educational stakeholders and mathematicians worldwide (Mesa et al., 2012). This interest stems from geometry's vital role in helping learners develop their capacity for critical thinking and problem-solving. (Clements, 2004). Despite its importance, geometry, particularly circle theorems, is perceived by students as difficult due to its complex and abstract nature (Aidoo-Bervell, 2021). WAEC (2013-2018) report cited in Segbefia (2020) exhibited that candidates struggle with geometry problems, including cyclic quadrilaterals, chord theorems, and circle theorems. These reports also reveal that many candidates faced challenges in solving problems related to exterior and interior angles of polygons (WAEC & GOG, 2019; WAEC, 2017). The May/June 2013 and 2016 WASSCE results showed that students often avoided questions requiring the application of geometric concepts.

Additionally, the 2017 WASSCE report noted that most candidates struggled to apply the correct geometric theorems to determine certain angles (Ntow & Hissan, 2021). The Chief Examiner's reports (WAEC, 2020) further suggest that students' abysmal competence in geometry may result from teachers' neglect of circle theorems or their inability to use effective teaching methods. The teaching methods educators employ significantly impact student learning and skill acquisition (Oppong-Gyebi et al., 2023). Teacher-centered approaches often result in passive student involvement, hindering performance in mathematics (O'Connor et al., 2000). This view is supported by Ntow and Hissan (2021), who describe this traditional teaching style as inadequate for fostering understanding. Lim (1992) observed that teachers often rush through circle theorems without ensuring students grasp the foundational concepts.

To enhance student engagement and understanding, it is essential to implement effective teaching strategies. The inductive teaching approach, which encourages students to observe, explore, and draw conclusions, has been shown to improve achievement and retention (Segbefia, 2020). Hidalgo (2017) suggests that inductive teaching aligns with psychological principles, promoting deeper understanding and knowledge retention through natural cognitive processes. Several studies have investigated the efficacy of an inductive approach to teaching in educational settings (Prince & Felder, 2006; Acharya, 2017; Mensah-Wonkyi & Adu, 2016; Rahmah, 2017).

However, these studies did not specifically focus on circle theorems. Segbefia (2020) examined the impact of inductive teaching but did not assess the specific difficulties students faced in solving circle theorems. This study aims to fill these gaps by investigating the specific difficulties students face in solving circle theorems and the impact of inductive teaching on students' performance in solving circle-theorem problems.

In order to achieve the aforementioned goals, the following research questions were employed:

1) What are the difficulties students face in solving circle theorem problems?
2) What effect does the inductive approach to teaching have on students' academic performance in solving circle theorem problems?
3) What are students' perceptions about the inductive teaching method in circle theorem instruction?

To answer research question two, a hypothesis was set as;

Ho: Inductive teaching has no significant effect on students' academic performance in solving circle theorem problems.

1.2. Literature Review

1.2.1. Constructivist Theory of Learning

Inductive teaching is deeply rooted in constructivist learning theory, which views knowledge as actively constructed by learners through experiences, interactions, and reflection rather than passively received (Chuang, 2021). Constructivism positions students at the centre of learning, emphasizing inquiry, exploration, and pattern recognition. In mathematics, particularly circle theorems, this approach enables students to discover relationships and formulate generalisations, thereby fostering critical thinking, conceptual understanding, and problem-solving (Selçuk, 2010; Williams, 2017).

This framework draws from the philosophies of Dewey, Piaget, and Vygotsky, who argued that learning occurs through inquiry, social interaction, and assimilation of new knowledge into existing schemas. Piaget (1952) highlighted the processes of schema development, assimilation, accommodation, and equilibration, while Vygotsky (1978) emphasised social constructivism, peer collaboration, and the Zone of Proximal Development, in which students advance through scaffolding. Thus, in inductive lessons, group discussions, guided questioning, and shared exploration help learners co-construct knowledge. Inductive teaching aligns with various learner-centered models such as inquiry-based, problem-based, and project-based learning (Prince & Felder, 2006). These methods encourage students to engage in problem-solving, hypothesising, and deriving rules from examples, supported by visual aids and real-world contexts. While initial confusion may arise, this process helps eliminate misconceptions and build a more resilient understanding (Prince & Felder, 2006; Sjøberg, 2010). The teacher's role in this approach is to facilitate rather than transmit knowledge, providing scaffolding, creating aligned learning environments, and intervening only when necessary (Biggs, 2003; Hailikari et al., 2022). By engaging students in meaningful tasks, inductive instruction cultivates broader competencies such as collaboration, creativity, self-assessment, and flexibility, which are essential for lifelong learning (Bernard & Dudek-Różycki, 2019). The mediating role of teacher communication reported by Akendita et al. (2025) aligns with social constructivist views, which emphasize the teacher's role in scaffolding learners' meaning-making processes. In the inductive teaching of circle theorems, effective communication is essential in guiding learners from specific cases to generalized geometric principles. Ultimately, constructivist-based inductive teaching makes learning mathematics more active, student-driven, and enduring. In teaching circle theorems, it transforms abstract content into meaningful exploration, equipping students with deeper conceptual mastery and transferable problem-solving skills.

1.2.2. Empirical Review

Empirical studies reveal that circle theorems, though fundamental in geometry, present considerable challenges to students due to their abstract and visual nature. Learners often struggle to connect theoretical concepts with practical applications, leading to poor mastery (Ntow & Hissan, 2021; Suglo et al., 2023). This difficulty is compounded by reliance on traditional, rote-based teaching methods that emphasize memorisation rather than understanding (Özerem, 2012; Aidoo-Bervell, 2021). Such challenges are not unique to Ghana but are reported globally (Khurshid & Ansari, 2012). Contributing factors include inadequate teacher preparation, limited teaching aids, poor prior knowledge, and psychological barriers such as math anxiety (Fabiyi, 2017; Swindal, 2000).

In response, several studies have examined alternative instructional approaches. Mensah-Wonkyi and Adu (2016) found that inquiry-based teaching enhanced conceptual understanding and motivation compared to conventional methods. Similarly, Segbefia (2020) showed that inductive teaching significantly improved achievement in circle theorems, with no gender differences. Bruno (2013) reported that diagrammatic instruction facilitated easier conceptualisation and application of geometric concepts. Ansong et al. (2021) identified student difficulties and recommended connecting circle geometry to real-life contexts. Another study, Akendita et al. (2025), found that teacher effective communication significantly mediated the relationship between students' mathematics interest and mathematics performance. This finding suggests that instructional strategies that actively engage learners and are clearly communicated are more likely to yield positive academic outcomes. The implication for the current study is that inductive teaching, which relies on guided examples and student discovery, requires effective teacher communication to enhance students' performance in solving circle theorem problems. Ameen et al. (2022) demonstrated that a Problem-Based Instructional Strategy (PBIS) greatly improved performance, again without gender differences. More recently, Akendita et al. (2024) confirmed that socio-constructivist mathematics teaching positively influenced achievement, with mathematics self-efficacy mediating this effect.

The evidence underscores the need for student-centered, active learning strategies such as inquiry, inductive, problem-based, and socio-constructivist approaches, which consistently outperform traditional methods. Integrating visual aids, technology, and real-life contexts can further enhance understanding, while strengthening self-efficacy remains a key pathway to improved performance.

2. Methods

The study was guided by a pragmatic paradigm and employed a sequential explanatory mixed-method design. A quasi-experimental pre-test–post-test control group design was used, with 84 second-year SHS students selected from a population of 388 using convenience, proportionate stratified, and purposive sampling. G*Power software determined the sample size (42 in the experimental group and 42 in the control group). Data were collected using achievement tests (pre-test and post-test) adapted from WASSCE questions and a semi-structured interview. The instruments were validated by experts and shown to be reliable (Cronbach's alpha = 0.88). To ensure trustworthiness, the study adopted Lincoln and Guba's (1985) criteria of credibility, transferability, dependability, and confirmability. Credibility was enhanced through triangulation, prolonged engagement, member checking, and peer debriefing, ensuring that findings accurately reflected participants' views. The experimental group received inductive instruction, while the control group was taught using the conventional method over four weeks, both covering the same circle theorems. Data analysis involved descriptive statistics, t-tests, and thematic analysis. Ethical considerations, including informed consent and confidentiality, were duly observed.

3. Results and Discussions

3.1 Results

3.1.1 The Difficulties Students Face in Solving Circle Theorem Problems

The difficulties students face in solving circle-theorem problems were analysed and reported in this section. Given that this is an experimental study, the analysis of students' difficulties was based on their responses to the pre-test, specifically the theory section (Sec B), which was designed to assess students' prior knowledge and uncover the specific challenges they encountered before any instructional intervention was introduced. Using the pre-test results allowed for a more accurate identification of students' natural difficulties with circle theorems, independent of any instructional support. It also serves as a baseline for understanding the areas where students require the most support. Students' difficulties were grouped into five broad categories: reading and comprehension difficulties, poor visualisation of geometric relationships, misapplication of theorems/properties, lack of procedural fluency, and computational difficulties. Descriptive statistics for the difficulties described above are presented in Table 1.

Table 1 - Difficulty Types Encountered by Students in Solving Circle Theorem

Difficulty Type	Frequency	Percentage
Reading and comprehension	25	13.67
Poor Visualisation of Geometric Relationships	50	27.32
Misapplication of Theorems/Properties	64	34.97
Lack of Procedural Fluency	31	16.94
Computational Difficulties	13	7.10
Total	183	100

Source: (Field data, 2025)

Presented in Table 4 above is the distribution of difficulty types identified from students' responses to the pre-test on circle theorems. In total, 183 difficulty types were recorded. Among these difficulty types recorded, misapplication of theorems (64) was the most frequently occurring difficulty, representing 34.97% of all difficulties identified. This means that many students, even after recalling the right theorem, either misapply it or fail to meet the conditions necessary for applying specific geometric principles. Moving on from the foregoing difficulty, the second most frequently reported difficulty was poor visualisation of geometric relationships (50), accounting for 27.32%. Challenges in this category included failing to recognise angle properties and relationships between chords, tangents, or arcs, and difficulties interpreting or reasoning from the given diagrams. Students were also lacking procedural fluency; this difficulty was observed 31 times (16.94%). Students in this category were able to recall relevant theorems, but they had challenges with the organisation of solution steps, pointing to a gap in students' strategic competence and fluency in applying mathematical processes. Furthermore, reading and comprehension difficulties were observed for 25 instances (13.67%). These cases involve students misreading the question, misunderstanding key terms, or failing to identify the specific mathematical task required of them. Lastly, the least frequent difficulty observed was computational difficulty, which occurred 13 times (7.10%). These included mostly basic arithmetic or algebraic errors made during carrying out the correct procedures. While not as conceptually serious as other errors, they nonetheless affected the accuracy of final answers. Following this statistical summary, a content analysis was conducted to further interpret the nature of these difficulties, drawing on specific examples from student work. Excerpts are shown below.

3.1.2 Reading and Comprehension

Reading and comprehension difficulties, as used in this study, refer to students' struggles to understand or interpret the language of the mathematics problem, particularly the instructions, terminology, or contextual information presented. These challenges are not necessarily tied to a lack of mathematical knowledge, but rather to weaknesses in processing written information or connecting the language of the task with relevant mathematical concepts. An excerpt is shown in Figure 1 below.

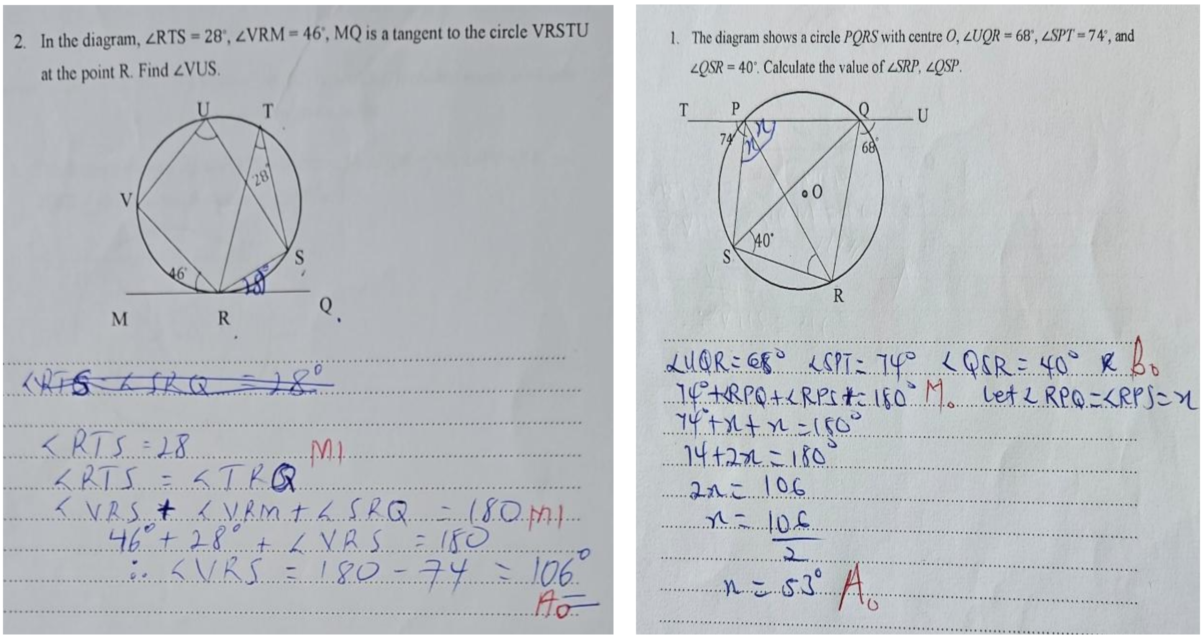

Figure 1. Exhibition of Students' Difficulty in Reading/Comprehension

From Figure 11 (Q2), student 43 was able to write down all the information needed to solve the question. Even though an unjustified assumption that $\angle RTS = \angle TRQ$ was made without any geometric property supporting it. While the mathematical approach was sound, the student calculated $\angle VRS$ =106° instead of the requested $\angle VUS$. This indicates a reading comprehension error where the student misread which angle was being asked for in the question. Per the solution provided by student 23 (Q1), $\angle SPR$and $\angle RPQ$ was assigned to the variable x, which geometrically does not add up. Besides the angles ($\angle SRP$ and $\angle QSP$) required the student to compute, which was not even addressed, reflecting a lack of comprehension of both the question and geometric concepts.

3.1.3 Poor Visualisation of Geometric Relationships

Poor visualisation of geometric relationships, as identified in this study, refers to students' difficulty in *seeing* and *making sense of* the spatial structure of a diagram. This includes challenges in interpreting geometric representations, recognising how points, lines, angles, arcs, and chords are connected, and mentally constructing how different parts of a figure relate to one another. In circle theorem problems specifically, students with weak visualisation often fail to identify key relationships embedded in the diagram—for example, not noticing which angles subtend the same arc, confusing a tangent with a secant, or overlooking that radii to points of tangency form right angles with the tangent line. As a result, they may select an incorrect theorem or apply a correct theorem to the wrong elements.

These difficulties are commonly linked to procedural thinking, in which students attempt to recall formulas or theorems mechanically without first analysing the diagram's structure. Instead of "reading" the figure for geometric cues (e.g., equal arcs implying equal angles, cyclic quadrilaterals implying supplementary opposite angles), students may focus only on surface features—such as the presence of a circle—without establishing the intended spatial relationships. Poor visualisation can also appear when students cannot keep track of multiple relationships simultaneously (for instance, when a single angle is related to both a chord and an arc), or when they struggle to extend lines, imagine auxiliary constructions, or interpret implicit information that is not explicitly labelled.

An example of this issue is shown in Figure 2, where students' work demonstrates difficulty identifying the relevant geometric elements and connecting them to the appropriate circle-theorem relationships. This leads to incomplete reasoning, incorrect angle identification, or the use of unrelated theorems, even when the necessary information is present in the diagram.

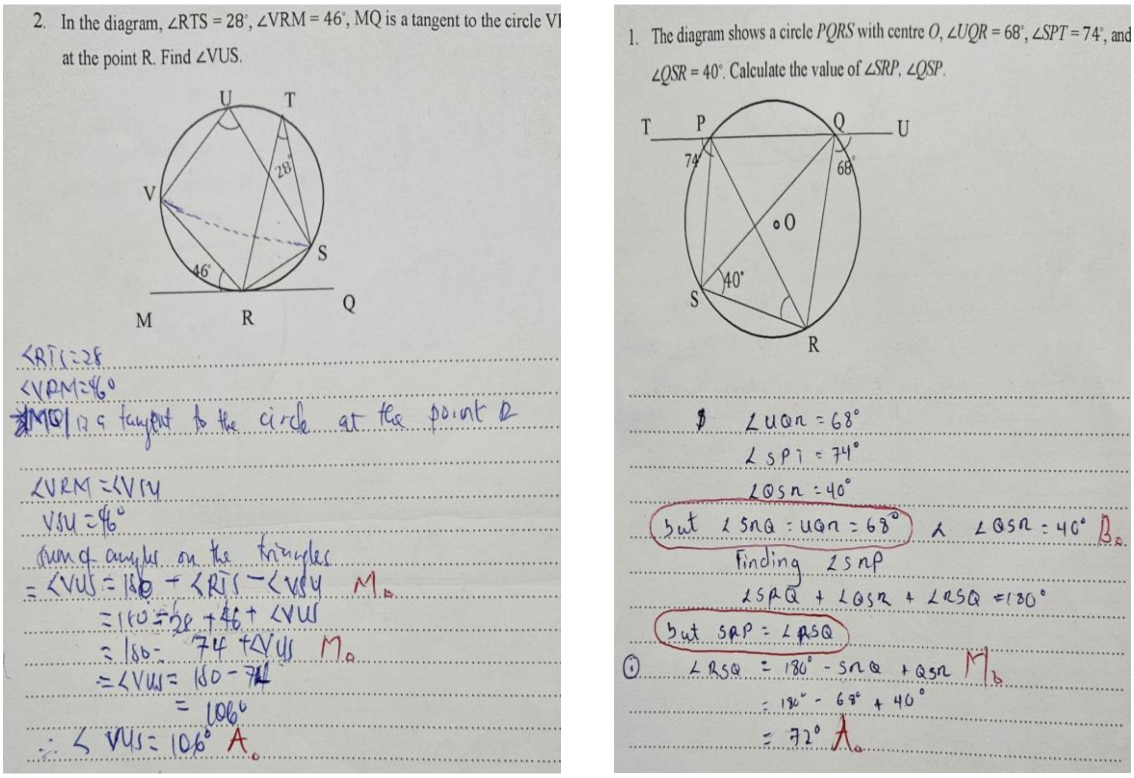

Figure 2. Exhibition of Students' Poor Visualisation of Geometric Relationships

As exhibited in Figure 2 above, Student 41 approached Q2 by rightly recognising $/MQ/$ as a tangent; however, as a result of the student's poor visualisation of geometric relationships could not recognise that $\angle SRQ = \angle RTS$, drawing from the alternate segment theorem which would have helped him proceed with the solution process. Instead, the student equated $\angle VRM$ to $\angle VSU$,which geometrically is wrong. Also the student went on finding the value for $\angle VUS$ by subtracting $\angle RTS$ and $\angle VSU$ from 180o, however, these angles were not related as they are not located in a same triangle further suggesring that the student has a poor visualization of geometric relationships. In a similar vein, the response given by Student 7 in her quest to solve Q1 indicated poor visualisation of geometric relationships. The student stated in her solution "but $\angle SRQ = \angle UQR = 68°$" , vewing the two said angles as alternate angles without noticing that $/SR/$ and $/QU/$ are not parallel. The student further established that $\angle SRP = \angle RSQ$,viewing the two angles as being equal even though their point of intersection was not the centre of the circle.

3.1.4 Misapplication of Theorems/Properties

Misapplication of theorems or properties as used in this study is the situation where students, in their attempt to use geometric rules or relationships, misapply them. This most often happens because students either misunderstand the conditions under which the theorems applied will be considered valid or because they confuse one theorem with another. The occurrence of this difficulty in circle geometry is when one is able to recall and apply a theorem, but in the wrong context, misidentifying the elements involved, or applying a rule that does not fit a given diagram. This difficulty reflects a partial understanding of a learnt theorem or an attempt to recall a remembered procedure without fully getting its meaning, as such it could be considered as being usually conceptual. While the student might appear to be engaging with the problem meaningfully, the student's reasoning is often mathematically incorrect due to incorrect assumptions or mismatched properties, even though, student might appear to be engaging with the problem meaningfully. Exhibition of such difficulty is depicted in Figure 3 below.

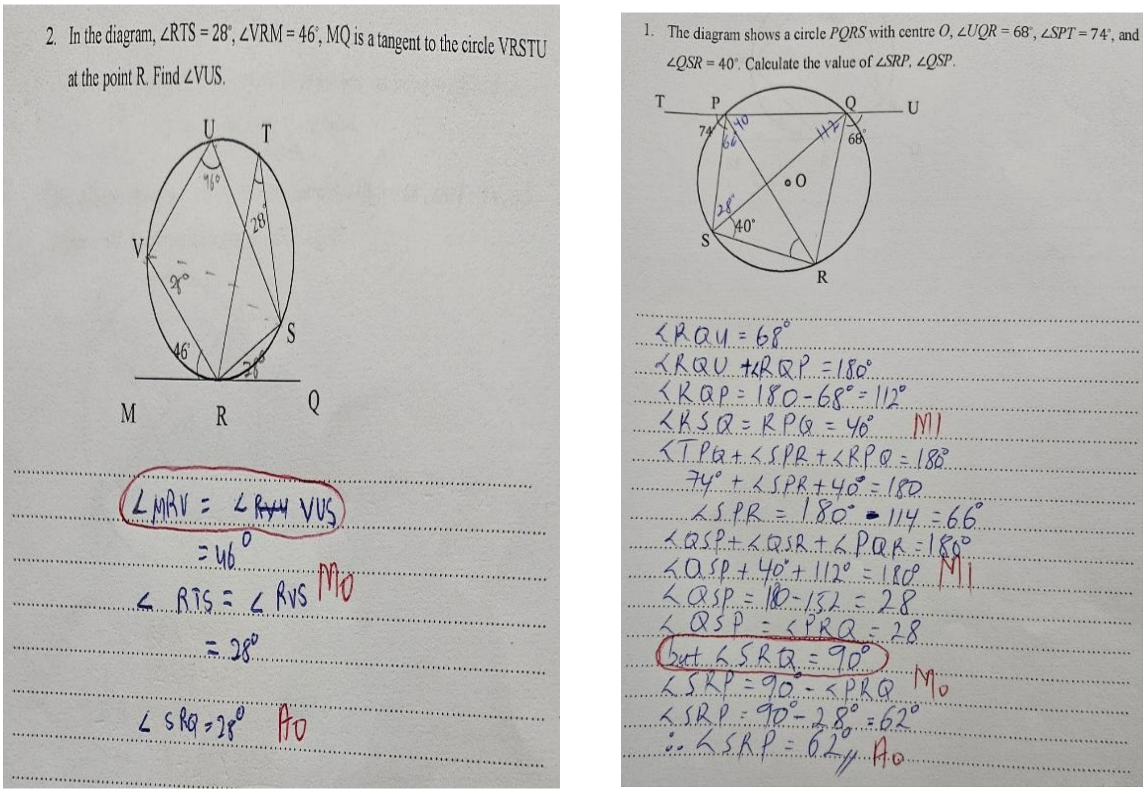

Figure 3. Exhibition of Students' Poor Visualisation of Geometric Relationships

From the response given by Student 13 to Q1, it was observed that the student accurately related $\angle SRQ$ to $\angle STR$ as revealed from the diagram; however in the actual workings the student equated $\angle MRV$ to $\angle VUS$ viewing them to be equal which per the theorem and geometrically is wrong. The student did this as a result of misapplying the exterior and opposite interior angle theorem, which prevented him from answering the question correctly. Similarly, for Q2, the response given by the student depicted a fair knowledge of the concepts of circle theorems, as observed from the start of his (Student 63) presentation. Unfortunately, the student recalls the angle in a semicircle theorem, in other words, the angle subtended by a diameter; however, it was applied in the wrong context. The learner viewed the chord $/SQ/$ as a diameter, which led him to conclude that $\angle SRQ = 90°$, which resulted in getting the rest of the solution wrong.

3.1.5 Lack of Procedural Fluency

Students' difficulty in carrying out a series of mathematically sound steps in the context of solving a problem is what is referred to as lack of procedural fluency in this present study. Not always does this difficulty in geometry, and particularly in solving circle theorem questions, stem from a lack of understanding of individual theorems. Rather, it reveals itself when students struggle to make use of multiple steps to coherently follow a clear solution pathway. That is where students need to make use of one idea in geometry to build on the next. Sometimes, students, instead of following each step carefully, skip parts. When that happens, some important details or parts are left out. Some students even report the right answers without necessarily showing how they worked it out, making it sometimes difficult to follow their reasoning. An excerpt of such difficulty is shown in Figure 4 below.

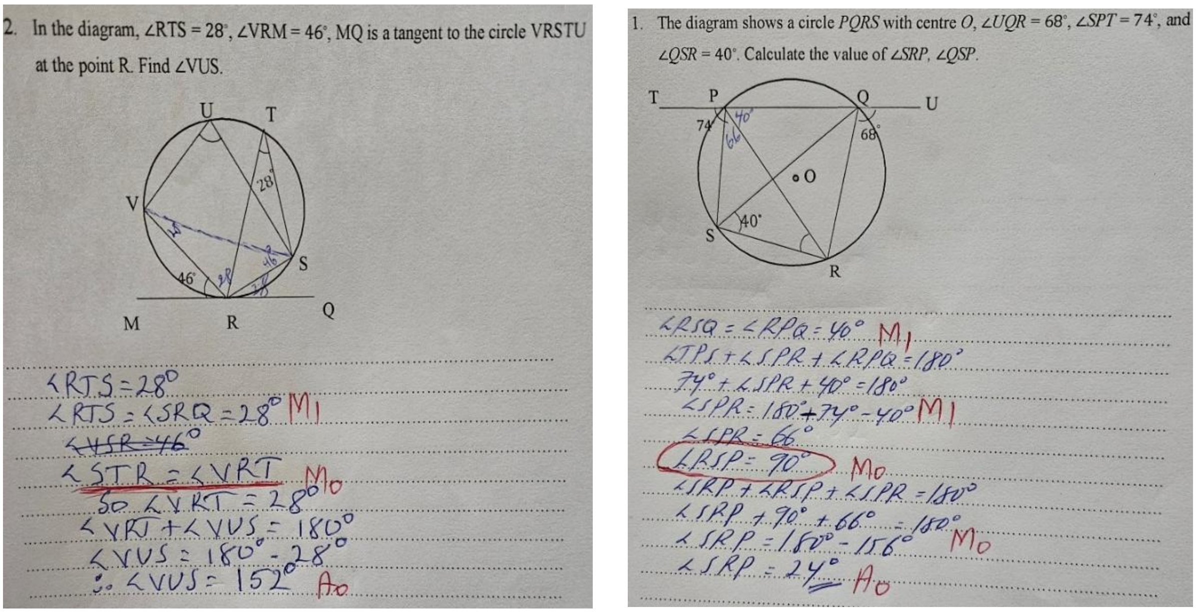

Figure 4. Exhibition of Students' Lack of Procedural Fluency

In response to answering Q2, Student 39 provided a solution, shown in Figure 14 above. In the solution, the student rightly recalled and made use of the alternate segment theorem by correctly equating $\angle RTS$ to $\angle SRQ$. However, the student applied a wrong procedure by stating $\angle STR = \angle VRT$, drawing from the concept of alternate angles without noticing that, $/VR/$ is not parallel to $/TS/$ rendering the rest of the solution wrong. The student even stated $\angle VRT + \angle VUS = 180°$. Here, the student tried making use of the opposite angles in a cyclic quadrilateral theorem; however, the procedure was not right, as the angle opposite to $\angle VUS$ is $\angle VRS$ and not $\angle VRT$. Moving on, Student 71 attempted Q1 by providing the solution shown in Figure 14 above. The solution process started well until the student reached the part where he stated "$\angle RSP = 90°$", which geometrically is wrong. The chord $/RP/$ was treated as a diagonal, and such the angle $\angle RSP$ was taken to be $90°$, which is not so. So here, students, although rightly recalled the theorems, the procedure utilised was not mathematically acceptable.

3.1.6 Computational Difficulties

Computational difficulties are the mistakes students make when carrying out basic arithmetic or algebra steps while solving problems. In circle theorem questions, this often happens after they have applied the right theorem but end up with a wrong answer because of calculation or simplification errors. Rather than conceptual, these mistakes are usually mechanical. A student may be able to identify the right relationships, make use of the right values, as well as choose the right formula, but still lose marks because of slips such as sign errors, wrong operations, or ignoring the order of operations. Even though these errors don't always show a lack of understanding, they still affect performance and can hide otherwise correct reasoning. A few such errors were found, and an example is shown in Figure 5 below.

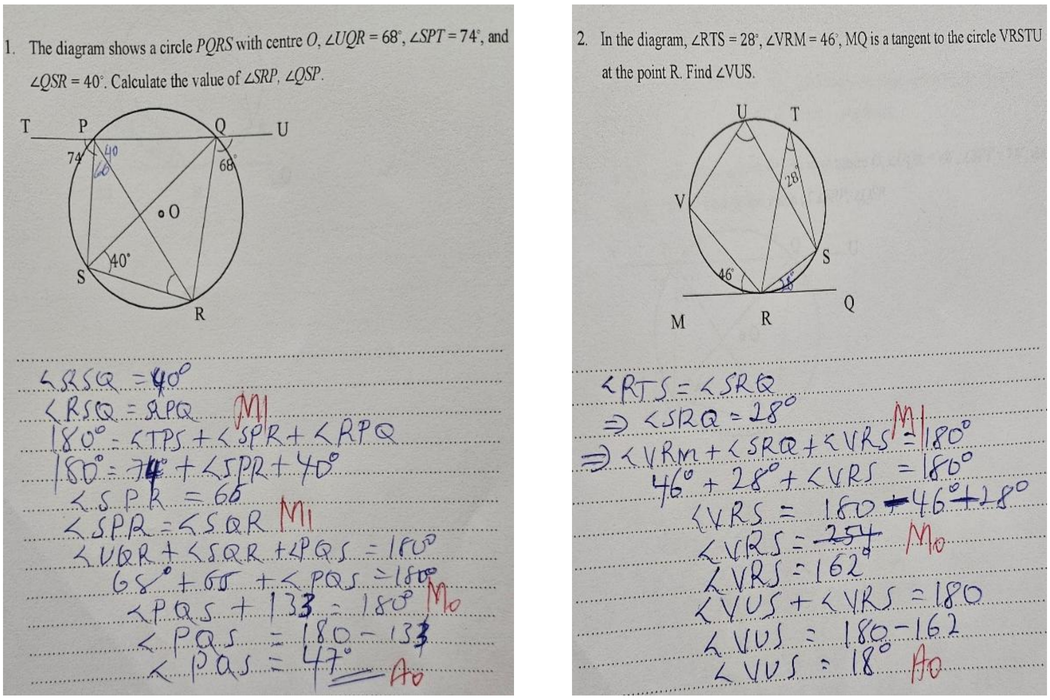

Figure 5. Exhibition of Students' Computational Difficulty.

Student 55's response to Q1, as shown in Figure 15 above, reveals a flaw in the mathematical computation. The solution process started well, as everything indicated that the student knew what he was doing, theorems were recalled and applied appropriately; however in stating the value for $\angle SPR$. The student wrote 66, looking more like a 65. Subsequently, in the next four steps, the sum of the angles $\angle UQR$ and $\angle SQR$, was computed as 133 instead of 134. The reason could be an oversight, as the student mistakenly added 65 to 68 instead of 66, which, as a result, led to getting the final answer as 47° instead of 46°. In a similar vein, in Q2, Student 13 started well with the right theorem by equating $\angle RTS = \angle SRQ$. However, in computing for $\angle VRS$A computational difficulty was observed as the student had the value for the said angle as 162. Upon critically analysing this flaw, it was found that the student treated -28 as $+28$, resulting in getting an incorrect value for $\angle VRS$, leading to wrong subsequent solutions.

3.1.7 The Impact of Inductive Approach Teaching on Students' Academic Performance In Solving Circle Theorem Problems

Test for Normality Assumptions

Before running the main statistical test, to know how the effect of the inductive teaching method impacts students' performance in circle theorem problems. It was considered essential, drawing from standard practices to check whether the data met the normality assumption. According to Pallant (2020), parametric tests such as the independent-samples t-test work best when the data are roughly normally distributed, especially with smaller sample sizes. This helps ensure that p-values and confidence intervals are trustworthy and that the conclusions are valid (Razali & Wah, 2011). In this study, students' scores on the pre-test and post-test were measured on an interval scale, and since participants were randomly assigned to control and experimental groups, normality testing was needed before proceeding. Two tests were used for this purpose: the Kolmogorov–Smirnov and the Shapiro–Wilk tests. The results are shown in Table 2 below

Table 2 - Normality Test Across Groups and Tests

Test	Groups	Kolmogorov-Smirnova			Shapiro-Wilk		
		Stat.	df	Sig	Stat.	df	Sig
Pre_Test	Control	.15	42	.02	.944	42	.04
	Experimental	.18	42	.00	.937	42	.02
Post_Test	Control	.17	42	.00	.944	42	.04
	Experimental	.10	42	.200*	.959	42	.13

*. a lower bound of the true significance.

As exhibited in Table 2 above, the current study considered the Shapiro-Wilk test, given that, is widely acknowledged as more suitable for moderate sample sizes ($n = 42$). The results shown in Table 2 indicate a deviation from normality for both the pre-test and post-test across the various groups. With the exception of the experimental group's post-test scores, where the p-value (.13) exceeds the 0.05 threshold, the rest, specifically the control group's pre-test (p = .04) and post-test (p = .04), as well as the experimental group's pre-test (p = .02), all show statistically significant deviation from normality. Levene's test for equality of variance for the two groups was also conducted (see Table 3).

Table 3 - Group's Levene's Test for Equality of Variance

Tests	Groups	F	Sig
Pre-Test	Control	1.23	.27*
	Experimental		
Post-Test	Control	2.33	.13*
	Experimental		

As depicted in Table 3, both the pre-test and post-test scores yielded F-values of 1.23 (p=0.27) and 2.33 (p=0.13) for the two distinct groups, respectively. This indicates that, in both cases, the p-values were greater than the 0.05 threshold, indicating that the assumption of equal variance was met and hence not violated (Field, 2009).

Drawing from the above two tests discussed above, although the normality assumption was not fully met, the sample sizes for both groups are equal (42) and sufficiently large (>30) to justify the application of the central limit theorem (Field, 2009; Elliott & Woodward, 2007). Informed by Ghasemi and Zahediasl (2012), in the case where group sizes are equal with no extreme outliers, t-test and other parametric tests are strong to mitigate departures from normality assumptions. Again, referencing Field (2013) and Kim (2013), normality assumption can be violated in the context of conducting a t-test, provided it is run with bootstrapping for participants greater than 30. In view of this, the data were considered suitable for parametric analysis; in particular, independent and paired samples t-tests were performed alongside boostrapping with the aid of SPSS version 27.

Comparison of Pre-Test Scores Between Groups

To assess whether both groups had comparable levels of understanding of circle theorems prior to the instructional intervention, an independent samples t-test was conducted using the pre-test scores of the control and experimental groups. This comparison was necessary to ensure that any observed differences in post-test performance could reasonably be attributed to the intervention rather than pre-existing disparities in knowledge. The results of the analysis are presented in Table 7 below.

Table 4 - Independent Sample T-test Results for the two Groups' Pre-test Scores

Groups	N	Mean	SD	t-stats.	df	Sig
Control	42	9.02	3.61	1.06	82	.29
Experimental	42	8.31	2.48			

Table 4 shows that the control group had a mean pre-test score of 9.02 (SD = 3.61), while the experimental group had 8.31 (SD = 2.48). An independent-samples t-test with bootstrapping was performed to determine whether this difference was significant. The result, (t(82) =1.06, p =.29), shows that there was no meaningful difference in the pre-test scores of the two groups. This means both groups began at about the same level in their understanding of circle theorems before the intervention. As a result, any difference seen in the post-test can be linked more clearly to the teaching method used.

Within-Group Comparison of Pre-test and Post-test Scores

To determine whether there was a significant improvement in students' performance after the intervention within each group, paired samples t-tests were conducted for both the control and experimental groups. This was done to assess whether the students made statistically significant progress over time. The results of the analysis are presented in Table 5.

Table 5 - Paired Samples T-test Results for Pre-test and Post-test Scores

Test	N	Mean	SD	t-stats.	df	Sig
Control Group						
Pre-test	42	9.02	3.61	-9.02	41	.00
Post-Test	42	11.36	3.07			
Experimental Group						
Pre-test	42	8.31	2.48	-17.38	41	.00
Post-Test	42	13.81	3.38			

Table 5 shows that the control group had a mean score of 9.02 (SD = 3.61) on the pre-test and 11.36 (SD = 3.07) on the post-test. A paired-samples t-test confirmed that this increase was significant (t(41) = –9.02, p < .001), indicating that the control group improved over time. Likewise, the experimental group, which received instruction through the inductive method, reported clear improvement. Their mean score increased from 8.31 (SD = 2.48) on the pre-test to 13.81 (SD = 3.38) after the intervention. This change was also significant (t(41) = –17.38, p < .001). Overall, the results indicate that both groups improved, though the experimental group made greater gains.

Between-Group Comparison of Post-Test Scores

To determine whether there was a statistically significant difference in students' performance between the control and experimental groups after the intervention, an independent samples t-test was conducted on the post-test scores of the two groups. This analysis was conducted to establish whether the instructional method had a differential effect on students' understanding of circle theorems. The results are presented in Table 6.

Table 6 - Independent Samples T-test Results for Post-test Scores

Groups	N	Mean	SD	t-stats	df	Sig
Control	42	11.36	3.07	-3.48	82	.00
Experimental	42	13.81	3.38			

From Table 6, the control group had a post-test mean score of 11.36, while the experimental group scored higher with 13.81 out of 20. The difference between the two groups was 2.45. The t-test result ($t = -3.48$, $p < .001$) shows that this difference is statistically significant, since the p-value is below .05. In simple terms, the experimental group, who learned circle theorems through the inductive method, did better than the control group. The effect size, Cohen's $d = -0.76$, also indicates a moderate to large effect, suggesting that the inductive teaching approach (intervention) made a real difference in students' performance.

<u>Students' Perceptions About the Inductive Teaching Method in Circle Theorem Instruction.</u>

To explore students' views on the inductive teaching method, a semi-structured interview was held with five students from the experimental group who had experienced the approach during the lessons. The school runs five different programmes, so one student was chosen from each programme to take part. This section presents what came out of those interviews. The analysis was guided by Braun and Clarke's (2006) thematic analysis steps, which include getting familiar with the data, coding, looking for themes, reviewing them, naming them, and finally writing up the report. Since each student represented a different programme, they were given anonymous codes (Student A–E). Their responses were then examined for patterns in areas such as learning outcomes, engagement, motivation, confidence in mathematics, collaboration, and how useful they found the teaching method. From the analysis, five main themes were identified, and each is supported with direct quotes from the students to give a clearer picture of their perceptions.

<u>Theme 1: Deeper Conceptual Understanding through Discovery</u>

From what the five students said, the inductive approach made circle theorems easier to understand, not just something to memorise. Instead of being told the rules and proofs right away, they were asked to look at diagrams, measure angles, and notice patterns on their own. One student said, *"Before, I was just memorising the formulas and theorems, but now I actually know where they come from and why they work. You (researcher) used more examples and gave us time to find things ourselves. I was happy because it made things clearer"* (Student A). Another added, *"It felt like we were building the ideas from scratch. Our teacher usually just gave us the theorem and explained it, but the way you (researcher) taught made me feel like I discovered the rule, not just copied it"* (Student D). This way of teaching shows that students learn better when they are involved, which is also what constructivist theory says. The guided discovery made it easier for them to see how angles, chords, and arcs are connected in a circle, even though those ideas can seem hard at first. Hiebert and Grouws (2007) also note that real understanding in maths is more likely when students take charge of making sense of it, instead of just listening. Students themselves pointed out that this approach helped them really see the concepts, not just memorise them. As one student put it, *"I now understand the reason behind the rule, so even if I forget it in exams, especially WASSCE, I can figure it out again"* (Student E).

<u>Theme 2: Increased Motivation and Engagement</u>

In the interviews, students often talked about feeling more motivated in these lessons. For most of the students, the way and manner in which the concepts were taught was appreciated because formulas were not just given to memorise. They were encouraged to explore, look for patterns, and work things out on their own. Based on what they expressed during the interview session, that made the lessons felt active rather than the usual routine. Words used by students during the interview included "interesting," "fun," etc., when they described how the class atmosphere was during the instructional period. They also mentioned that the simple tools in their mathematical set, such as protractors, as well as the circle diagrams used as teaching and learning materials, helped them see how the ideas worked in practice. It was not just drawing shapes and naming parts. Angles were measured by students, while results were compared with classmates, and from this, they tried to state a rule from what they saw. The hands-on work made the learning feel real. As one student said, *"Hmm, Sir, for me, I usually do not like mathematics at all, but this made me want to come to class because it was different and I felt like I could do it"* (Student C). Several students said they started to look forward to class. Instead of avoiding mathematics, they were curious about what they would be asked to figure out next. One put it plainly: *"We were involved in the lesson, not just sitting down. I liked that. It made me want to come to the mathematics class"* (Student E). Taken together, the responses suggest that the inductive approach raised students' interest. Being part of the discovery made the lessons more engaging and enjoyable. Another student explained, *"Ok, Sir, for me, I prefer the other reading subjects because most of the time I find mathematics lessons boring with too many rules and formulas, but the lesson this time was not like that. I was interested. I wanted to be part of finding the answer myself"* (Student B). Motivation in mathematics learning is widely acknowledged as essential, especially when it comes to abstract topics like geometry. So when it happens that students feel they are helping to build the knowledge themselves, their intrinsic motivation tends to increase (Boaler, 2015). These positive feelings can also have lasting effects on engagement (Deci & Ryan, 2000).

Theme 3: Confidence in Problem-Solving and Reasoning

Another point that came up in the interviews was that students felt more confident solving problems after being taught with the inductive method. It was admitted by most of them that, before the intervention, circle theorem questions felt intimidating and confusing. For some of them, a decision concerning not attempting a circle theorem-related question in the WASSCE has already been made. However, after the concepts were delivered to them through a series of guided activities, there was a change of mindset as they expressed more control of their learning. For them, the process not only improved their understanding but also gave them a stronger belief in their ability to solve problems on their own. As Student B explained, *"After going through the drawing of circles, lines, taking measurements and coming to know the theorems, I felt I could solve the questions on my own. I can say those activities that went on have given me the confidence because now I know where the theorem came from."* Students compared this with how the topic had been taught before, when the teacher simply wrote the theorems on the board and asked them to memorise and apply them. They admitted that, in those earlier lessons, they often struggled to apply the rules correctly because they did not fully understand them. Being part of the discovery process, however, gave them a clearer picture of why and how the theorems worked. Student C remarked, *"Alright, Sir, currently I can say that, even if I forget the theorem, I can still find the answer by reasoning it through. However, it was to be the first, I would be stuck and could not continue."* Student D expressed a similar view: *"Getting to know the theorem made more sense than just memorising. I could see why the angle was that way, and that made me feel like I actually understood it. So now I can say I am not scared of seeing a new question because I am sure I can apply the concepts to solve it."* Student E highlighted the shift in mindset more directly by expressing that, *"What I do not like solving are circle theorem questions because what I know is that I will get them wrong. But this time around, I was more sure that I could rightly answer such questions."* These reflections show that the inductive approach did not just help students perform better but also boosted their confidence concerning their own reasoning. They felt more ready to handle unfamiliar problems because they had worked out the ideas step by step, rather than relying only on memory. In this way, the approach helped them move from dependence to independence. Confidence, as these students' comments show, is not just about getting the right answer, but also how they approach problems in class; that is, whether or not they try, how long they persist, and whether they believe they can find a solution.

Theme 4: Collaborative Learning and Peer Interaction

How useful it was for students to work in groups during the lessons was another point that emerged from the interviews I had with the selected students. For most of them, being able to talk with classmates, share ideas, and solve problems together was a great opportunity, as through that, learning became easier and also more enjoyable. The group tasks gave them the chance to think through the work, especially when they were trying to see patterns in circle theorems or figure out how to solve certain questions. One student said, *"Sir, for me, as someone who usually prefers to do things alone, working in groups helped me a lot because I got to learn different ways of solving the questions. Sometimes another person would notice something I didn't"* (Student E). From these comments, it is clear that working with peers allowed students to see different sides of a problem. Sometimes, one student would be able figure something out and explain it in a way the others had not thought of yet. Through this, the lesson felt more like a shared effort instead of an individual struggle. Student B also said, *"Explaining to others helped me understand better. It forced me to think about what I was doing."* This shows that when students tried to explain their own reasoning to others, it actually helped them to understand more deeply themselves. Another student added, *"It was easier to ask my group than to raise my hand in class. I wasn't afraid to be wrong"* (Student D). For many students, speaking in front of the whole class can be intimidating, especially if they are unsure of their answers. Small group work made it easier and safer for them to ask questions and join in the discussion. This seemed to encourage more participation overall. As one student explained, *"Sometimes when I didn't get something, my friend would explain it in a simple way. It was like having another teacher beside me"* (Student C). From these responses, it seems clear that group work within the inductive teaching method was not only helpful but sometimes even more effective than when the teacher explained everything directly. Students felt supported, encouraged, and involved, because they could depend on one another as well as the teacher.

<u>*Theme 5: Challenges in Abstraction and Need for Support*</u>

Issues concerning the use of the inductive teaching approach were also highlighted by students, especially during the initial stages of the lessons. Some said that when they were first asked to look at the diagrams and try to find patterns by themselves, it was not easy. Since they were more used to being told the rules straight away by the teacher, figuring things out on their own felt unfamiliar and even a little frustrating. One student explained, *"Sometimes I find it a bit hard to notice the pattern immediately. So, I wanted the teacher to just explain it"* (Student A). This shows that while discovery learning has clear benefits, it also poses its own difficulties. Not every student will pick up the idea right away, and this can cause confusion or even self-doubt. In mathematics, where students often expect direct answers, that can be especially challenging. Another student added, *"It was confusing at first. You have to think hard and try again. It takes time to get the idea"* (Student E). A similar concern was expressed by another student who shared that, *"For me, I think providing a little help would make it better. Like I am not saying give us the answers o, but something to point us in the right direction"* (Student C). It must be noted that, inductive way of learning concepts is not always straightforward. It takes time, patience, and a willingness to let students struggle a little as they work things out on their own. That struggle may feel uncomfortable, both for the learner and sometimes even for the teacher watching it happen. Yet when handled with the right balance of support, it often leads to a stronger, more lasting grasp of ideas. Students clearly gain from figuring things out for themselves, but they rarely do so in isolation. They still need guidance that points them in the right direction without taking over the thinking. This is very much in line with Vygotsky's idea of the zone of proximal development, which emphasises that learners progress best when they are supported just enough to build on what they already know.

3.2 Discussion

3.2.1 The Difficulties Students Face in Solving Circle Theorem Problems

The study first looked into the difficulties students face in solving circle theorem problems using pre-test data collected before any inductive teaching intervention. This preliminary analysis served to identify learners' baseline challenges and to gauge the areas in which they most needed support. Analysis of the pre-test responses revealed five categories of difficulty, namely, misapplication of theorems, poor visualisation of geometric relationships, lack of procedural fluency, reading and comprehension errors, and computational mistakes, with differing frequencies as summarised in Table 4. Importantly, misapplication of theorems accounted for 34.97% of all instances, followed by poor visualisation (27.32%), lack of procedural fluency (16.94%), reading and comprehension (13.67%), and computational errors (7.10%).

Reflecting on the high rate of misapplication of theorems is an indication that although many learners possess the ability to recall the theorems, when it comes to applying them correctly in practice, they struggle. For example, a number of students applied the angle-at-the-centre theorem when dealing with an inscribed angle, or assumed a quadrilateral was cyclic without first establishing the conditions. These errors are not surprising if we consider what has been reported elsewhere. Students in South African secondary schools were also noted to be able to correctly state a theorem, but applied them in context where the conditions did not hold (Ubah & Bansilal, 2019). Likewise, Suglo et al. (2023) found that students preparing for the West African examinations had difficulty deciding when a theorem was valid, often confusing the requirements that made each one applicable. These studies suggest that recall on its own is not enough. Simply put, successfully recalling a theorem does not guarantee comprehension. As a result, I argue that exhibition of such difficulty is an indication of memorised knowledge not yet connected to deeper reasoning about the geometry.

Poor visualisation of geometric relationships came out as the second most frequent difficulty, making up about 27% of the errors. A number of students seemed to have trouble noticing and reasoning with the visual features of the diagrams. For example, some overlooked equal angle markings, others misread arcs or chords, and quite a few could not form a clear mental picture of how the different parts of the diagram related to one another. Rodd (2010) points out that geometry depends heavily on visualisation, and without that skill, learners are likely to miss the hidden relationships that are essential for solving problems. Several other studies (Mensah et al., 2022; Mulligan, 2015; Newcombe et al., 2019; Novita et al., 2018) make a similar case, showing that students who cannot mentally manipulate diagrams usually struggle with circle theorems. In the present study, it looked as though many students worked in a rather mechanical way. They picked out individual angles or chords but did not link these observations to the theorems that would help them move forward. This pattern suggests that their thinking was still around Van Hiele Level 1, or perhaps just moving into Level 2, where properties are recognised but not yet tied together into a deeper conceptual understanding (Bonyah & Larbi, 2021).

Making up 16.94% of the difficulties recorded, the lack of procedural fluency further highlighted the depth of students' reasoning gaps. It was observed in several cases that learners could identify the appropriate theorems, but organising them into a consistent sequence of steps is where the struggle is. Some skipped important steps, made assumptions without explanation. For some of them, the final answer for test items was provided without proper justification. This finding is consistent with Brijlall and Abakah (2022), who observed that many students struggle to connect individual reasoning components into a well-structured argument, which in turn makes their written solution weak. Similarly, per the report given by Barut and Retnawati (2020), students, while being able to understand the requirements of a theorem, were found not to be able to develop and carry out a clear sequence of reasoning, thereby undermining the accuracy and coherence of their final answers.

Although less common, reading and understanding difficulties accounted for 13.67% of the difficulty types. These challenges were likely due to misconceptions associated with the problem and fundamental requirements, such as understanding what angle and what measure to target. Misuse of mathematical language, particularly in geometry, as argued by Lin and Yang (2008) and Kwadwo and Asomani (2021), can be a challenge for learners who do not understand the terminology or intricate sentence constructions associated with the discipline.

Circle geometry, for example, relies on such precise vocabulary as 'subtend,' 'tangent,' 'intersecting chords,' and the like. Students who use these terms without adequate understanding often misuse theorems or possibly omit crucial parts of the question posed. In this study, instances of reading errors appeared when students presented solutions that were in no way related to the task at hand; this resulted from applying unrelated theorems to an incorrectly interpreted question. These types of difficulties, while not frequent, go on to demonstrate how language comprehension relates to conceptual and visual understanding.

Finally, accounting for the least among the difficulties observed was computational difficulties (7.10%). These included small arithmetic or algebra mistakes, such as adding instead of subtracting, even after the correct steps were followed. While these errors say less about students' understanding of concepts, they still affect the final correctness of answers. Per Mensah et al.'s (2022) assertion, such mistakes can be reduced provided concepts are clearer and problems are well-structured; in that case, students can be in a better position to judge whether their answers make sense. Some students in this study correctly solved some problems; however, some marks were lost due to computational slips. This is an indication that, aside from getting the conceptual understanding of circle theorems, accuracy in working out the answers still mattered.

Suggestion from these findings is that students' difficulties in circle theorem problems are rooted more in meaning and reasoning as compared to than in arithmetic. From a theoretical perspective, the pattern reflects the Van Hiele model of geometric understanding. Many students in the sample seem to operate at Level 1, where they recognise properties, or at Level 2, where they can make informal deductions. However, few appear to reach Level 3, which involves deductive reasoning. Because of this, students are often able to recall theorem names and basic properties but struggle to apply them in flexible ways or to draw conclusions from geometric relationships. This may explain why misapplication and weak visualisation were particularly common.

3.2.2 The Effect of the Inductive Approach to Teaching on Students' Academic Performance in Solving Circle Theorem Problems

Evidence concerning how the inductive teaching impacts students' circle theorem problem-solving was provided by the post-test results. Improvement in the performance of both groups was noted, which is an indication that the concepts of circle theorems were better understood. However, the experimental group, which received the intervention (inductive teaching), outperformed those in the control group who were taught the conventional way. Referencing the pre-test scores, the two groups started at a similar level, but the experimental group achieved a higher mean score in the post-test (13.81 out of 20) compared to the control group (11.36). Confirmation by the independent samples t-test ($t = -3.48$, $p < 0.001$) rendered the difference found in performance to be statistically significant. The moderate to large effect size (Cohen's $d = -0.76$) suggests that the inductive method not only produced measurable statistical gains but also carried meaningful educational value.

Rather than receiving straightforward theorems and proofs, students in the experimental group were actively engaged and encouraged to observe patterns, draw conclusions, and develop generalisations from specific cases. As such, the improvement observed in the experimental group may be attributed to the intervention. Aligning with this method has to do with how naturally students develop conceptual understanding when guided through discovery (Prince & Felder, 2006; Abdullah et al., 2020). The hands-on, exploratory nature of the inductive method made students engaged in the teaching and learning process, which in turn helped them to understand the relationships and properties that exist within circle geometry, which may have translated into better test performance.

Aligning with the findings of the present study are studies that emphasised the value of active and discovery-based learning in mathematics. For instance, in a similar study conducted by Segbefia (2020), it was noted that students who were taught inductively demonstrated higher performance in the post-test as compared to those in the control group who were taught the conventional way. Not only was the performance of the experimental group higher than the control group, but the difference in performance was found to be statistically significant, pointing to the potential benefits of engaging students in guided exploration rather than passive absorption of information. Aligning with this perspective is the prior study conducted by Atta et al. (2015), who also discovered at the basic school level that learners exposed to concepts through the inductive approach performed statistically and significantly better than those taught deductively. Suggestions from their findings indicated that deeper learning and retention can be supported, provided instructors encourage students to derive mathematical principles on their own, rather than merely receiving them.

Furthermore, the findings support those of Acharya (2016) investigated how different approaches to teaching impact students' performance in geometry topics, of which the circle theorem is not an exception. From the report, teaching approaches which emphasise active student engagement in discovering patterns and establishing relationships yielded better academic outcomes as compared to the conventional approach, which is didactic in nature. Again, the report indicated that, while through the inductive teaching approach, students' conceptual understanding was enhanced, their problem-solving skills were also developed. Even though the inductive teaching approach was seen in the study as a catalyst for enhancing students' overall performance in solving circle theorem problems, the finding contradicts those of Rahman (2017) who conducted an experimental study involving junior high school students with inductive teaching as the intervention but at the end found no statistically significant improvement in students comprehension of mathematical concepts as well as their problem-solving abilities. Based on this, it can be said that the success of inductive teaching may depend on contextual variables such as the age of students, prior knowledge, instructional design, and teacher competence.

3.2.3Students' Perceptions about the Inductive Teaching Method in Circle Theorem Instruction

Valuable insights into how teaching inductively impacts students' understanding of geometric concepts were provided, referencing the feedback students gave during the interview session. Interestingly, the overall feedback received from students concerning the inductive teaching approach was positive. It was noted from the responses given by students that the inductive teaching made the lesson more engaging and effective as compared to the traditional teaching approaches their teachers have been using to teach them. The opinions expressed by students support the position that inductive teaching strategies not only enhance students' conceptual understanding but also promote positive dispositions towards mathematics learning.

Among the points raised by students, understanding the "why" behind the theorems, not just the "how", was consistently reported. According to the students, the approach was the one that facilitated them to "figure things out for themselves" or to "see the pattern," rather than simply memorising rules without context. Supported by the argument made by Hiebert and Grouws (2007) and Koskinen & Pitkäniemi (2022), the opinions as expressed by students are a reflection of meaningful mathematical learning, which can only be achieved when students actively are given the opportunity to construct knowledge. Discovering geometric relationships, according to the students, became memorable through guided tasks. It is noteworthy to say that these findings are not unique, as similar findings have been documented by other prior studies. This assertion is backed by Gholam (2019) and Ješková et al. (2016), who observed a statistically significant contribution of making use of inquiry-based and problem-based instruction as far as stronger retention of mathematical concepts is concerned, as compared to the traditional lecture-based approaches. Students who formed part of their study were actively engaged so far as knowledge construction is concerned, and in turn, performance was observed to be statistically significant on conceptual assessments. Further supporting the findings of the present study is Simamora & Saragih (2019), who argued that emphasised that deeper connections between abstract ideas and visual representations, a crucial skill in topics like circle theorems, are formed when students are provided with the appropriate guidance to discover rules and principles in mathematics on their own

Again, from the responses provided by students in the interview, it is obvious that students found the lessons more enjoyable and motivating than how they had been taught earlier before the intervention. It was described by most of the students that the lesson was "fun," "different," and "less boring," which I will say may appear superficial on the surface, but actually reflects a key emotional shift and how they engage with mathematics. Theoretically, as Deci and Ryan (2000) explained in their self-determination theory, intrinsic motivation os students increases when they feel competent, autonomous, and involved in the instructional process. How the inductive teaching approach was structured in the present study to met all these three needs of students. Not only were students solving problems, but doing so in a way that felt active and self-driven. Similarly, an observation reported by Siller and Ahmad (2024) and Barnes (2021) indicated that when students were engaged in exploratory and collaborative learning, significant improvement in their interest and enjoyment was seen. It can be deduced from here that students are more likely to see mathematics as relevant and approachable when they are encouraged to act as investigators rather than passive recipients.

Confidence as a key construct in mathematics learning was reported by students. It was shared by many that they felt within themselves to be able to approach, relate concepts to new problems and solve them following their exposure to the inductive teaching. At first, students were relying heavily on memorisation and rote learning; given this, it is arguable to say that this shift from memorisation to reasoning is an indication that the inductive method fostered into students a form of procedural confidence, where they felt they will be able to reconstruct the logic behind a solution not necessarily relying on memorised formulas (Segbefia, 2020). Theoretically, the argument made by Bandura (1997), which posits that students' belief in their own capabilities plays a central role in how they approach learning tasks, supports the finding of the present study. When small repeated successes are encountered by students, as is often the case of leaving students to explore, confidence that transfers to being able to approach and solve future problems is built (Ramadhani, 2018).

The essence of having the opportunity to collaborate with peers in problem-solving also featured prominently in the perception expressed by students concerning the inductive teaching. The opportunity to discuss problems with peers, exchange ideas, and justify their reasoning was appreciated by most of them. It is clear from this that this social aspect of learning geometry, situated in Vygotsky's (1978) sociocultural theory, particularly the notion of the zone of proximal development, where learners benefit most from tasks they can complete with the help of more knowledgeable peers or scaffolding from teachers (Fani & Ghaemi, 2011; Atta & Bonyah, 2023) cannot be undermined. For most of the students, the approach provided them with the comfortability to ask questions in small groups, which they would not have been able to do, supposing they should ask that same question in front of the entire class, suggesting that the inductive approach not only fostered academic growth but also created a more supportive classroom culture.

Despite the positive reaction received from the responses given by students concerning the inductive teaching, an aspect of the inductive teaching which, according to the students, was a challenge was reported. In particular, a moment of confusion during the starting stage of the instructional period was described. Students were not sure of what the goal was. This suggests that inductive learning can place a high cognitive load on learners, especially those not yet comfortable with open-ended problems, even though it supports deeper understanding. As noted by Kirschner et al. (2006), similar challenges were reported. The researchers argue that learners new to the approach may find it challenging to figure things out on their own without sufficient scaffolding. Based on this, the researchers suggested that for inductive and exploratory learning to be effective, it must be carefully structured and supported by timely teacher guidance.

4. Conclusions

Improving students' performance in circle-theorem problems requires a clear understanding of the difficulties they face, so potential alternative instructional approaches to address these challenges need to be explored. Students' difficulties with circle theorems were observed to primarily stem from conceptual misunderstandings, weak visual reasoning, and misapplication of theorems/properties rather than simple arithmetic issues. Students were observed not yet reasoning deductively with diagrams and relationships, which explains the frequent misapplication of theorems and the trouble interpreting figures.

The inductive teaching as the intervention, leading to significantly better post-intervention performance in circle theorems than conventional, lecture-focused methods, with an effect large enough to matter educationally, is an indication that the inductive teaching, inclusive of guided discovery, pattern-seeking, and hands-on construction, supports deeper learning and transfer to better academic performance. Students generally experience inductive teaching as meaningful and motivating. It supports autonomy, collaboration, and confidence. Some initial cognitive load is natural, so teacher scaffolding is important to help students organise tasks and make the intended generalisations. Overall, the results affirm that inductive teaching is a valuable pedagogical strategy for improving achievement and fostering positive learning experiences in geometry at the senior high school level.

The study recommends strengthening students' conceptual foundations in circle theorems through step-by-step, relationship-focused teaching, while prioritising inductive methods that encourage investigation, discussion, and generalisation. Teachers should receive training in inductive strategies, supported by low-cost teaching aids and digital tools. Emphasis should also be placed on improving students' language and diagram literacy, providing scaffolding to manage cognitive load, fostering collaborative learning, and integrating continuous assessment with timely feedback.

For future research, the study suggests replicating the intervention across diverse schools and regions to improve generalisability, and extending the intervention period to assess long-term effects. Research could also examine the application of inductive teaching in other mathematics domains, integration with technology, and the use of longitudinal designs to track performance and attitudes over time. Comparative studies across rural and urban contexts are recommended to explore how environmental factors influence instructional effectiveness.

Conflict of Interest

The authors declare no conflicts of interest.

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