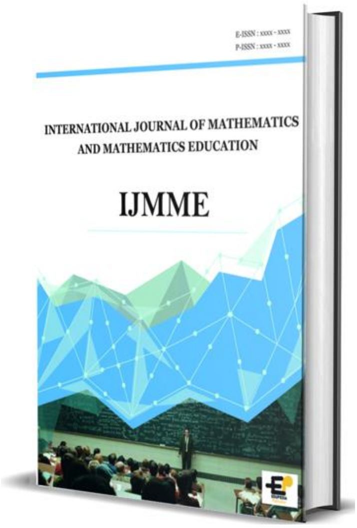

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Analyses of Pre-Service Teachers' Errors in Solving Algebraic Tasks in Ghana

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Abstract

This study analyzed prospective basic education teachers' errors in solving algebraic tasks using Newman's Error Analysis (NEA). The study employed an interpretivist philosophical paradigm with a qualitative case study design. Data were collected from 250 prospective mathematics teachers randomly selected from four colleges of education in the eastern region of Ghana using super item tests and unstructured interview protocols. Thematic analysis was employed to analyze the qualitative data. The study findings revealed that transformation and encoding errors among prospective mathematics teachers are the most prevalent errors. Other error types encountered included comprehension errors and process skills errors. In order to reduce aspiring mathematics teachers' errors, the study emphasizes the necessity of focused interventions and professional development programs to address transformation and comprehension errors in algebraic concepts.

Introduction

Mathematics is a bedrock of human progress and a powerful tool for comprehending the world and solving complex problems. Its applications extend across many fields, making it a crucial subject for education and research. Mathematics can explain the physical world in ways that are straightforward, systematic, and consistent (Ernest, 2014). Emphasizing the importance of mathematics in various domains encourages individuals to develop strong mathematical skills, fostering innovation and growth in society. Mathematics is one of the major pillars of education, especially at the basic and high levels of education (Frenzel et al., 2010), not only in Ghana but globally. Based on this, it has been made a compulsory subject for all levels of education in Ghana. So, placement into the university has been made a requirement. However, for someone to succeed in mathematics, algebra is seen to be one of the key curriculum categories offered to help students acquire mathematical knowledge and skills (Adjei & Oppong, 2024). Research shows that success in algebra is a factor in many other important student outcomes (Knuth et al., 2016). In this regard, algebra is the initial mathematics concept to be mastered by Ghanaian students before other mathematics concepts.

In Ghana, algebra is emphasized so much at the basic education level that pupils who can further their education to higher levels may not suffer in future mathematics-related courses. For our basic education pupils or students to do well in algebra, the teacher is believed to play a major role. This is why various educational stakeholders are concerned about the kind of teachers produced by the various institutions that train teachers for basic schools in Ghana.

According to Adjei & Oppong (2024), their study showed that preservice mathematics teachers have a solid understanding of the basic concepts and can make connections between various aspects of algebra. However, the preservice mathematics teachers struggle to apply their knowledge to new and complex situations or generalize their understanding. In other words, they have a good grasp of the foundational knowledge and can work with the information in a structured manner but face challenges in transferring that knowledge to novel or real-life contexts (Adjei & Oppong, 2024). Therefore, they continue by saying that participants' learning is limited to specific instances and cannot go beyond the immediate scope. This implies that prospective mathematics teachers' understanding of algebraic concepts is limited to basic knowledge and procedural skills. Hence, preservice mathematics teachers' curricula must be updated to address their challenges.

To identify the errors in Algebra that are most persistent and pernicious in predicting student difficulty on standardized test items, the present study analyses errors using NEA on preservice mathematics teachers' solving algebraic tasks. According to Sundarajan et al. (2020), one theory that can be utilized to identify the areas where students make errors when solving algebra problems is the Newman analysis approach. White's assertion that the Newman analysis-based error analysis has a high degree of credibility when it comes to identifying the errors that students make when completing algebraic tasks (Sari et al., 2018; Sugianto, Darmayanti et al., 2022; Sulistiawati & Surgandini, 2019). Teachers try to prevent pupils from making errors by providing scaffolding for those who make errors when answering questions. According to Sah et al. (2023), Vamsi et al., Kishore (2019), and Wood et al. (1976), scaffolding is a type of support that people give to pupils to help them solve difficulties. When students make errors, scaffolding is meant to help them identify their errors and make the necessary corrections (Ardıç & İşleyen, 2018; Sekaryanti et al., 2022; Sinaga et al., 2021).

For over ten years, students in Ghana's junior high schools have performed poorly on the Basic Education Certificate Examination in algebra and its applications (WAEC, 2011; 2012, 2013, 2014, 2015, 2017, 2019, and 2021). Following a thorough examination of these Chief Examiners' reports for mathematics, certain specific algebraic difficulties come to light. These include weak control when working with multiple variables in a single task, inappropriate bracket expansions (particularly those containing multiple terms and exponents), incorrect simplification of algebraic expressions, and inappropriate applications of the order of operations. These challenges seen in junior high school students provide proof that students are unable to demonstrate mastery of algebra and its applications, supporting the assertion that learners have several difficulties in handling algebra tasks

(Chaudhary, 2022; Chinnappan, 2010; Chow & Treagust, 2013; Matzin & Shahrill, 2015; Musi, 2023; Sultan & Artzt, 2011), which significantly affect their general performance in mathematics (Blume & Heckman, 2000; Ladele, 2013; Niringiyimana & Maniraho, 2023).

In recent times, the curriculum of the colleges of education has been switched from an objective-based mathematics curriculum to be in line with the standards-based mathematics curriculum of basic schools in Ghana to ensure college students have a strong grasp of fundamental mathematical principles and abilities that can be applied in real-life scenarios that could impact the students they would be teaching after college. This modification was made in response to the 2017 pre-tertiary curriculum review conducted by Ghana's National Council for Curriculum and Assessment (NaCCA), which aimed to reflect new and massive global trends in educational practices (Asante et al., 2024). The introduction of the new curriculum aimed to address several issues associated with the previous objective-based curriculum. These challenges encompassed issues like excessive content, limitations inherent to the objective-based curriculum, and shortcomings in the assessment system's ability to provide sufficient data for meaningful improvements in teaching and learning practices (Aboagye & Yawson, 2020). It focuses on establishing a well-structured and cohesive structure for cultivating mathematical literacy, problem-solving capabilities, and logical reasoning skills. The updated curriculum also strives to improve the teaching and learning of Mathematics. It emphasizes learner-centered teaching approaches leveraging ICT (Information and Communication Technology) as an instructional tool and ensuring all students have equitable resources to learn and succeed. Furthermore, it prioritizes an educational approach highlighting equity, equality, and inclusivity (Ghana Web, 2019, cited in Agbofa et al., 2023).

Before 2018, teachers in Colleges of Education in Ghana used to teach and assess Mathematics Content and Methodology as separate entities. However, a significant change occurred with the introduction of the Transforming Teacher Education and Learning (T-TEL) program. Tutors were now mandated to teach Mathematics Content and Methodology together. This shift was part of the T-TEL intervention, which designed a new curriculum in 2018 and actively ensured its implementation. Additionally, T-TEL emphasized conceptual and procedural knowledge in teaching and learning mathematics, as highlighted in a study (Salifu, 2021). For students to effectively grasp algebra, their teachers must profoundly understand the subject. Teacher training programs should offer the necessary content knowledge to help future educators comprehend algebra and, more specifically, variables. The research community in mathematics education has paid relatively little attention to prospective teachers' grasp of variables (Brown & Bergman, 2013).

In Ghana, students' performance on international benchmark assessments such as the Trends in International Mathematics and Science Study (TIMSS) has been consistently poor. This poor performance has raised concerns about the country's education quality. One significant factor contributing to this issue is the quality of teachers. Buabeng et al. (2014) state that teacher quality in Ghana is a major concern. The International Association for the Evaluation of Educational Achievement also highlighted in 2012 that poor quality teaching and teachers' inadequate content knowledge contribute to Ghana's low educational standards (Mullis et al., 2012).

Teachers, students, and materials are the three main components of teaching and learning. Students must be armed with information and advanced talents, and teachers must be competent and professional. Materials refer to the various resources, such as textbooks, videos, handouts, technology, etc., that teachers use to facilitate learning. Appropriate and high-quality materials are essential for effective teaching and learning as they help to explain and reinforce concepts and support students at different learning levels. Hence, teachers must be knowledgeable in their teaching field, especially algebra in mathematics at the basic level of education, since algebra is the linchpin to success in their high levels of education.

Introduced in early education and remaining a crucial subject at different educational levels, algebra is a key tool in mathematics (Jupri et al., 2014). Mathematical literacy largely depends on algebra (Malihatuddarojah & Prahmana, 2019; Ojose, 2011; Star et al., 2015). High school-level mathematics in Ghana builds upon the foundation of primary school education, with algebraic thinking as the foundation for progress to the next level in academics. Therefore, prospective basic education teachers must have a comprehensive understanding of algebra before starting their teaching career in order to overcome errors.

Errors are inaccuracies, mistakes, or deviations from correct procedures or understanding, often occurring during learning or performing tasks (Frese & Keith, 2015). It is a sign of assumptions students have about how things work. Therefore, it is crucial to educate aspiring elementary school teachers about the errors they make or might make when working through algebraic problems. This would enable the students or teachers to reflect on and fix their errors to prevent a repeat. According to Fitriani, Turmudi, and Prabawanto (2018), teachers must be aware of their students' errors when learning mathematics and offer proper and systematic solutions. They must also select the best strategy, model, or learning tool to help students understand concepts and develop problem-solving abilities.

These errors can be common among students when performing algebraic tasks, but they can also be specific to individual students. Lim (2012) identified twelve errors that students commonly make when simplifying algebraic expressions. These errors often result from various factors such as interference from new learning, difficulties with negative integers, misconceptions about algebraic tasks, and misapplication of rules. In essence, error analysis in mathematics education aims to pinpoint the nature and causes of students' errors, allowing educators to address these issues effectively and support students in mastering mathematical concepts and problem-solving techniques.

Students must go through several steps to arrive at their ultimate mathematical problem-solving solution. Polya (1973), cited in Abdul, Nur Liyana, and Malina (2015), identified four (4) steps for solving a mathematical problem: Understanding the issue, developing a plan, putting the plan into action, and reviewing the result of the first two steps. These techniques enable pupils to answer mathematical problems with ease. However, due to differing levels of thinking and issues understanding instructional methodologies, not all students can solve mathematical problems. This causes learners to commit a variety of errors and misunderstandings.

Statement of the Problem

Algebra gives us a new tool for comprehending real-world mathematical situations involving unknown values. Without algebra, the most crucial fundamental mathematics technique, nothing in science, technology, or engineering would be possible today. Algebra is the linchpin to success in mathematics because it plays a fundamental role in all fields of mathematics (NCTM, 2000; National et al., 2008; RAND Mathematics Study Panel, 2003). Research has indicated that elementary school students can acquire the basic concepts and skills of algebra and that a thorough and prolonged early algebra experience substantially impacts their mathematics comprehension (Blanton et al., 2015).

Even though algebra is essential for students' success in mathematics, other subjects, and real-life situations, it seems that students struggle to learn it, which has led to the general underperformance of students in the subject and mathematics as a whole. For instance, algebra is deemed tough because it is one of the most abstract threads in mathematics (Egodawatte & Stoilescu, 2015). According to Jupri and Drijvers (2016), transforming questions into mathematical symbolic problems and creating equations, schemes, or diagrams are the main challenges pupils face when completing algebra-related word problems. Sikukumwa (2017) highlighted other challenges, including problems with texts, unfamiliar settings, improper strategies, and a deficiency in skills for problem-solving. Due to their lack of algebraic skills, which are necessary for other mathematics subtopics, students cannot perform well on mathematics examinations.

According to Higgins et al. (2002), cited in Awuah (2018), errors students make are related to their inability to focus while solving mathematical problems, a lack of thinking skills, memory overload, and a failure to recognize some key aspects of a problem. This suggests that although students can understand a topic that has been taught, they often forget the steps involved, hence making errors when attempting to solve mathematical problems relating to that topic. According to Cangelosi et al. (2013), a learner may be stuck at a low level of growth for the linked notion when there are persistent errors. In this regard, there is a need to identify the errors in algebra which are most persistent in predicting student's difficulty in algebraic tasks and mathematics as a whole. This study uses Newman's Error Analysis (NEA) to explore and analyze algebraic errors in prospective mathematics teachers' solving algebraic tasks.

Theoretical Framework

Systematic and non-systematic errors are the two categories of errors (Awuah, 2018)—a systematic, deliberate error results from repeatedly providing incorrect information. Repeatedly using incorrect information throughout the process may result in correct answers, matching the incorrect information, making it extremely difficult to recognize systematic errors over time. Due to this, unless students are helped to become aware of them, these mistakes become persistent and are difficult for them to fix on their own. Inaccurate responses result from non-systematic errors, which are inadvertent and non-repetitive errors that happen frequently in calculations (Awuah, 2018). These mistakes are typically the consequence of carelessness and can be readily fixed by the students.

Additionally, Cheng-Fei (2012) distinguished between procedural, factual, and conceptual errors that students commit. He described factual error as the mistake students make due to a lack of mastery of a fundamental fact or the inability to recall a specific fact, procedural error as the failure of a student to follow the proper step or procedural in solving mathematical problems, and conceptual error as the mistake made as a result of poor understanding of a particular concept in mathematics. He continued by asserting that the curriculum materials, instructional design, and mode of delivery used in the mathematics classroom are all to blame for the mistakes above and ignorance, inattention, and carelessness. In order to ensure that instructions are followed correctly and answers are given, he advised teachers to recognize the errors and misconceptions that students regularly make.

In Newman's (1977) classification of errors, as cited in Fitriani, Turmudi, and Prabawanto (2018), there are five distinct groups of errors that can be analyzed using Newman's Error Analysis (NEA). This model of error investigation, initially proposed by Newman in 1977, has proven reliable and beneficial for mathematics teachers. Additionally, several other researchers, such as Allan (2010), Casey (1978), Clarkson (1980), and Effandi, Ibrahim, and Siti (2010), have concurred on the model's reliability. Newman's Error Analysis (NEA) offers a structured framework for examining the reasons behind students' difficulties with mathematical problems and provides a process to help teachers pinpoint where misunderstandings occur. NEA also provided directions for where teachers could target effective teaching strategies to overcome them.

According to (Newman, 1977), reading error refers to a student's inability to understand a mathematical problem and identify the words or symbols provided in the questions. After carefully reading the question, the second sort of inaccuracy occurs when students cannot comprehend or relate to the symbols, expressions, and issues provided. The third type of error, called transformation error, is when students cannot select the suitable formula, method, or property and relate it to the solution of a given problem. Process skill error once more. This is a process skill error when students cannot utilize the proper technique or operations or make an error during the procedures. Encoding error is the last. Encoding errors look into students' mistakes in generating and justifying or drawing inferences from the answers they have given.

According to a study by Trance (2013) that Abdul et al. (2015) quoted, 70% of participants committed errors in NEA-based comprehension and transformation. The study aimed to evaluate students' learning and performance of the algebraic idea. Participants were required to perform an exam based on NEA, and the questions required them to solve algebraic problems orally before demonstrating the computation methods. The study suggested that students should receive extra instruction on faults like understanding and transformation before tackling algebraic problems to reduce these errors because of the widespread failure of students on these concepts.

Therefore, it is crucial to educate aspiring elementary school teachers about the errors they make or might make when working through algebraic problems. This will enable the students or student teachers to reflect on and fix their errors to prevent a repeat. According to Fitriani et al. (2018), teachers must be aware of their students' errors when learning mathematics and offer proper and systematic solutions. They must also select the best strategy, model, or learning tool to help students understand concepts and develop problem-solving abilities.

Method

This study employed a qualitative case study design to identify algebraic errors among prospective mathematics teachers at colleges of education. Research that delves further into and examines issues in the real world is known as qualitative research. Qualitative research poses open-ended inquiries that are not readily quantifiable, such as "how" and "why." One of the advantages of qualitative research is its capacity to clarify human behavior patterns and processes, which can be challenging to measure (Cleland, 2017). This reveals nuances and underlying factors that quantitative data might not, offering an in-depth understanding of prospective mathematics teachers' algebraic errors. The sample included 250 prospective mathematics teachers from four randomly selected colleges. Participants were selected from four randomly chosen colleges, all of whom had completed at least one course in algebra. Data were collected using a super item test and unstructured interviews. Thematic analysis was used to categorize and analyze the errors according to NEA.

The Super Item test consisted of two (2) questions designed based on the SOLO taxonomy super item test format. The two questions were on the same topic (algebra), which validated the consistency and reliability of responses. The super-item test was constructed to ascertain prospective mathematics teachers' level of thinking in solving algebraic tasks concerning SOLO taxonomy. According to Yulian (2019), giving students an appropriate learning model enhances their learning achievement or outcomes. A good learning outcome indicates that the instructional objectives prepared were achieved. One such criterion is to construct test items from simple to abstract. Therefore, many researchers have approved and used the super item test (Collis et al., 1986; Lam et al., 1998; Wilson et al., 1988; Adjei & Oppong, 2024).

Five prospective mathematics teachers were asked to explain their working process during the Super Item Test through unstructured interviews. In this current study, the researchers opted to employ unstructured interviews to understand better how participants approached the Super item test. The unstructured interview enabled the researchers to identify the errors committed by the students in solving algebraic tasks. Unstructured interviews have an open-ended situation where greater flexibility and freedom are offered to both the interviewer and the interviewee in the contest of planning, organizing, and implementing the content of interview questions (Alshenqeeti, 2014). Participant observers tend to favor the utilization of in-depth, unstructured interviews, which are open-ended in nature. These interviews provide a basic framework of topics to be covered during discussions with participants.

Results

Details of the error types committed by the participants are shown in Figure 1 through to Figure 10 and alongside Table 1.0.

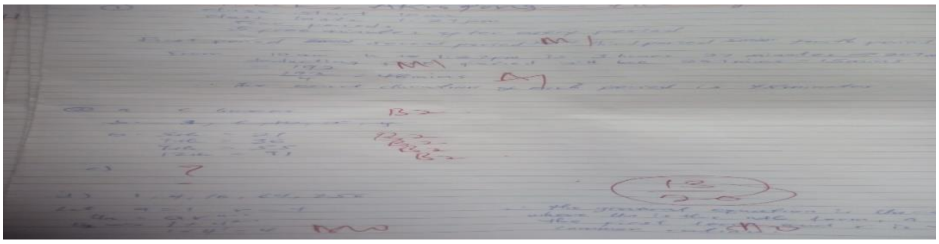


Figure 1. Obuobi's Sample test

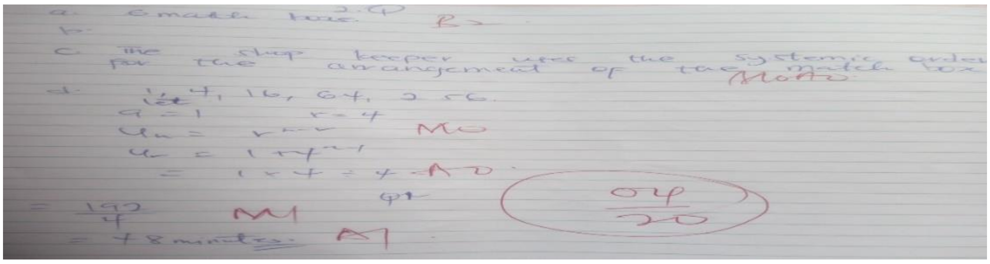


Figure 2. Delali's Sample test

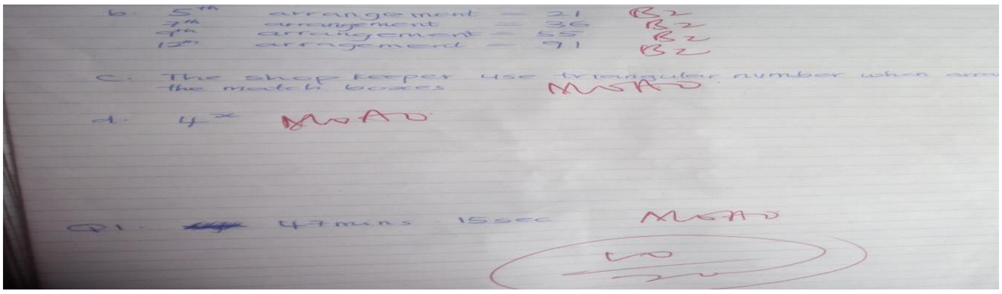


Figure 3. Raymond's Sample test

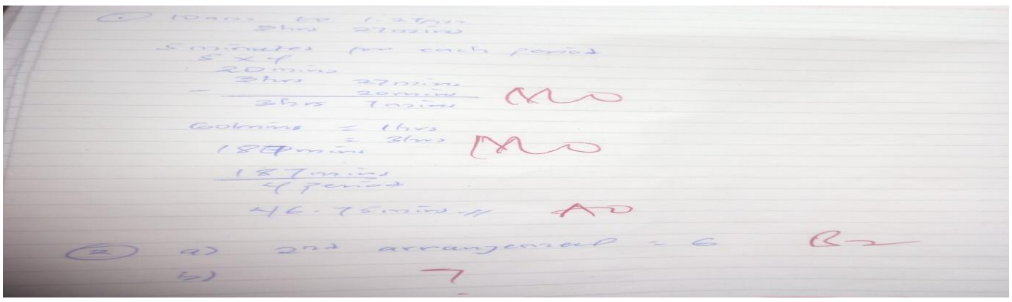


Figure 4. Gyampo's Sample test

Some students also had questions 2A and 2B wrong. These questions asked the participants to determine the number of matchboxes the shopkeeper uses in $2^{nd}, 5th, 7th\ etc$ Arrangements. Participants who had this question wrong commit comprehension and process skill errors. This is because if a student has difficulty understanding the terms used in the passage, the concept of different arrangements, or the significance of the information provided, they might not be able to answer the question accurately. Without a proper grasp of the content, they might not understand the context of the question and may struggle to make sense of what is being asked, hence committing comprehension errors. Moreover, to correctly answer the question, students must calculate or determine the number of matchboxes used in the second arrangement based on the information in the passage. This involves a basic counting operation. However, if a student has difficulties with counting, basic arithmetic, or logical reasoning, they might be unable to perform the necessary calculations accurately. This would result in a process skill error, where their inability to perform the required mathematical operation leads to an incorrect answer. Based on this, participants whose results are shown in Figures 4, 5, 6, and 9 commit comprehension and process skill errors. These errors were due to their inability to utilize the proper technique or operations or make errors during the procedures.

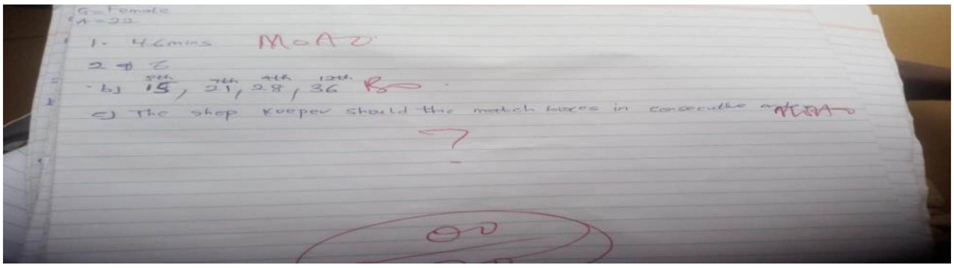


Figure 5. Malik's Sample test

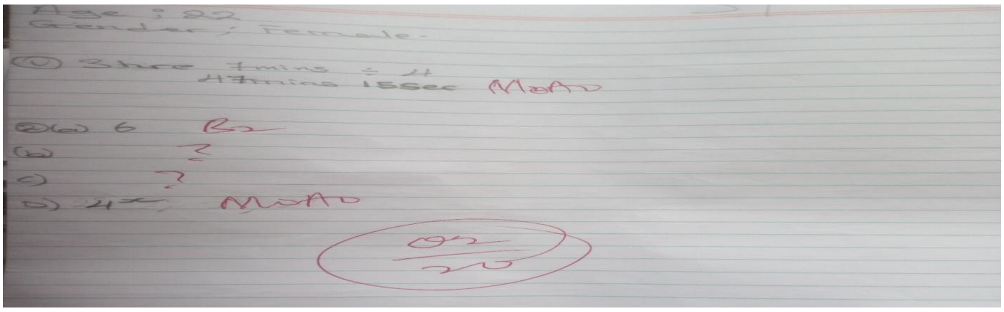


Figure 6. Kukua's Sample test

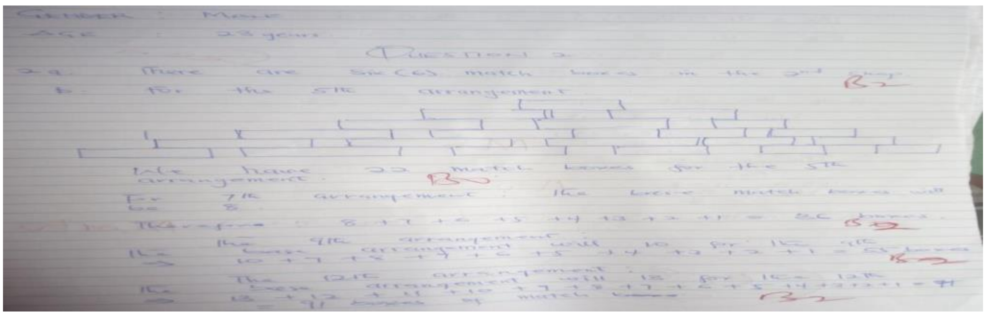


Figure 7. Kirie's Sample test

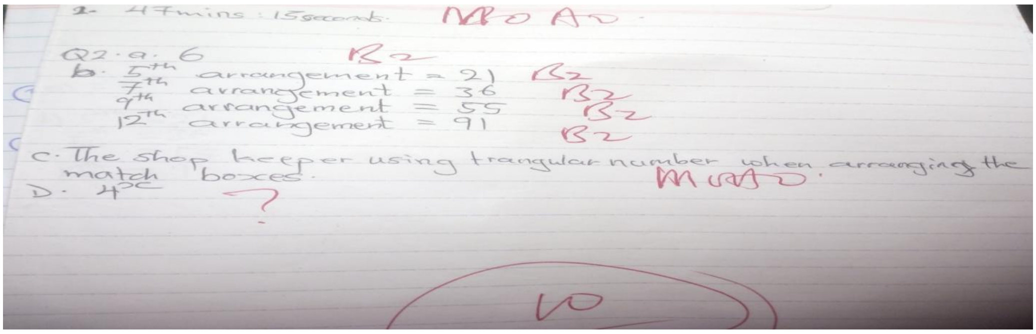


Figure 8. Puatuo's Sample test

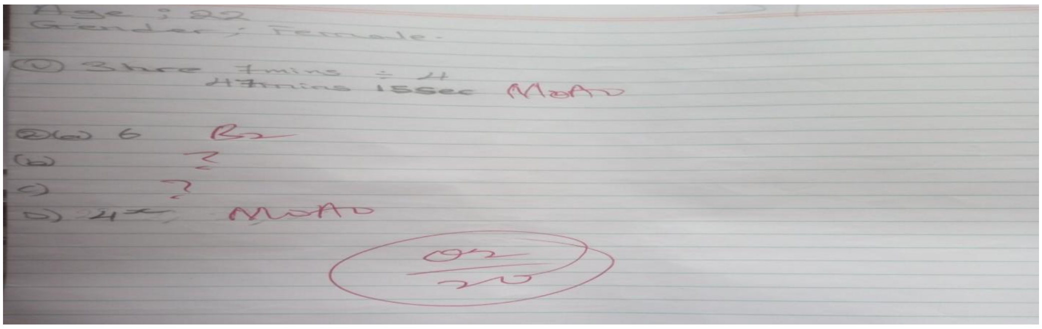


Figure 9. Lovette's Sample test

Figures 5, 6, 7, and 9 showed that the participants had question 1 wrong. Only Figures 2, 8, and 10 had question 1 correct. These individuals were said to be committing comprehension and transformation errors because question 1 is a straightforward question that provides a scenario and asks for a specific calculation (the duration of each period). If pre-service teachers get this question wrong, it indicates a failure to comprehend the information provided in the scenario or a misunderstanding of how to perform the required calculation. Pre-service teachers could not comprehend or relate to the symbols, expressions, and issues provided. Again, pre-service teachers could not select the suitable formula, method, or property and relate it to the solution of a given problem. On that note, both comprehension and transformation errors occurred.

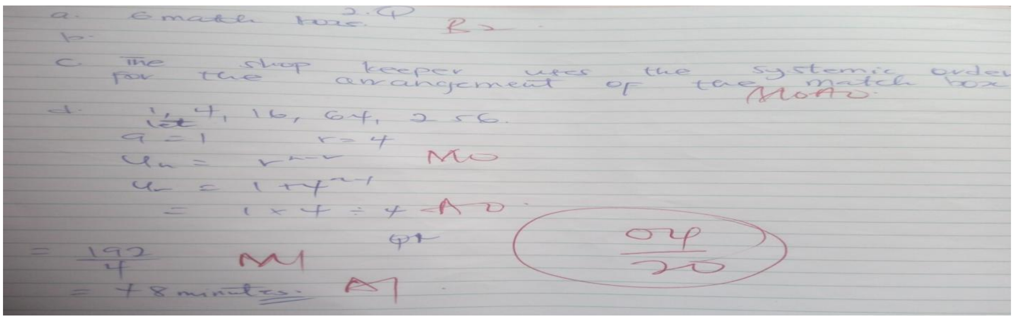


Figure 10. Addo's Sample test

In question 2C, the participants were asked to write a mathematical rule for the shop keeper to use when arranging the match boxes. Participants who had these questions wrong commit different kinds of error ranging from transformational error, comprehension error, process skill error and encoding error. These types of errors are illustrated in Figures 1 through 10, where each figure represents a different type of error made by participants. This indicate that, a participant may commit more than one error in each question.

The final question (2D) asked participants to develop a mathematical equation to generalise the arrangement. This is where the participants were to extend their knowledge to a different level of understanding. Assuming the shop keeper needs to arrange tin tomatoes and the number of tin tomatoes required for the respective patterns are 1, 4, 16, 64, 256, …… most students had this question wrong and committed different types of errors in regard. However, comprehension error was the major error committed at this level; though there were transformational, process skill and encoding errors. There were some other participants who do not attempt the question at all. These types of errors are illustrated in Figures 1 through 10, where each figure represents a different type of error made by participants. This indicate that most pre-service mathematics teachers struggled in generating and justifying or drawing inferences of the answers they had given which led to encoding error.

After assessing the participants' test results, further analysis was conducted using frequencies and percentages to quantify the number of errors committed by the participants. Each of the participant's response was rated on 100% since the fact that one person commit error 1 does not mean he or she cannot commit error 2. One person can commit more than one type of error hence each response is rated based on the total number of responses collected. Though there were five error types identified in the literature, reading error was not considered in the analysis because the researcher accept that the participants already can read and may not commit such errors regarding reading. As a result, reading error was not considered in the analysis. Details on the error type committed by the participants is shown in Table 1.0.

Table 1. Error type committed (*N = 250*)

Error	**Type**	**College A**	**College B**	**College C**	**College D**	**No. of errors committed**	**Percentage**
1	Reading Error	0	0	0	0	0	0.0
2	Comprehension Error	27	20	45	57	149	59.6
3	Transformation Error	49	53	39	48	189	75.6
4	Process Skills Error	20	12	48	18	98	39.2
5	Encoding Error	37	26	35	17	115	55.2

Source: Survey data (2023)

According to the analysis conducted on prospective mathematics teachers in selected schools, the errors committed by the teachers were categorized into different types: comprehension, transformation, process skills, and encoding errors. From participants' responses, it was discovered that there were also 149 comprehension errors, constituting 59.6% of the total errors. This indicates that many errors were caused by the prospective teachers' lack of understanding or confusion regarding algebraic concepts.

It was also demonstrated that 189 transformation errors were observed, making up 75.6% of the total errors. This indicates that most errors occurred while the teachers attempted to perform transformations or manipulations in mathematical problems. At the same time, 98 process skills errors were identified, accounting for 39.2% of the total errors. This also suggests that many errors arose from deficiencies in applying appropriate problem-solving techniques or mathematical procedures. There were also 115 encoding errors, constituting 55.2% of the total errors. This implies that many errors occurred while the teachers transferred mathematical information or calculations from one form to another.

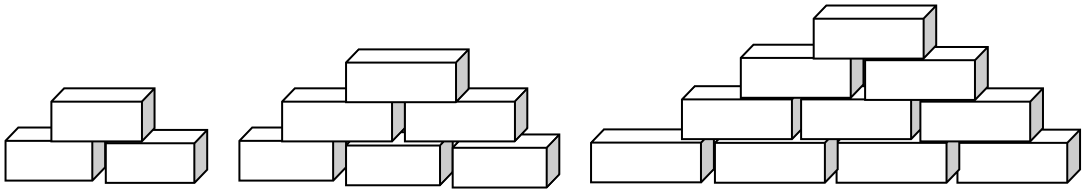
In summary, Newman's Error Analysis of prospective mathematics teachers' errors in selected schools indicates that the most prevalent types of errors were transformation errors (75.6%), comprehension errors (59.6%), and encoding errors (55.2%). This suggests that the prospective teachers struggled the most with manipulating and understanding mathematical expressions after reading them and transferring mathematical information accurately. Process skills errors (39.2%) also contributed to the overall error profile, highlighting areas where the teachers faced challenges in understanding algebraic concepts, applying problem-solving strategies, and accurately interpreting mathematical problems or questions.

Interview Response

Having collected data on participants using the super item test, further probing was done using an unstructured interview to inquire from selected participants who had challenges writing the test. The purpose of the interview was to help clarify the error types the participants committed while solving the algebraic tasks.

The participants' responses to the test showed that they mostly had challenges answering question 2. Therefore, some individuals were selected to explain what made it difficult and why they could not solve the questions.

The questions were based on the scenario “in a provision shop, the shopkeeper organizes various items in a shop. On each shelf, items like matchboxes are often placed orderly so would-be buyers or shopkeepers can easily locate them. They are also arranged so that removing them will be easy”. From this scenario, the follow-up questions were asked based on how the matchbox is shown in *Figure 1.10* below.



1^{st} $nd2^{nd}$ 3^{rd}

Figure 11. Question Structure

Participants were asked how they got the six matchboxes in Question 2A. This question asked, "How many matchboxes did the shopkeeper use. 2^{nd} arrangement?" Participants are expected to count the number of matchboxes used in the arrangement. It does not involve any requisite calculations or explanations. Nonetheless, most of the students had it wrong. According to participants,

> *Since the first arrangement has two matchboxes at the base and the second one has three at the base, I decided to count them. I see some patterns from the matchbox. I can see that in each arrangement, the base increases by one. However, I do not know why I could not do it well and had the answer wrong (R3). I did not get six matchboxes. I seemed to deviate, so I did not know what I was doing there. I know I had it wrong (R5).*

Based on the participant's responses, it was observed that they were attempting to determine the number of matchboxes in each arrangement based on the pattern they observed. They noted that the number of matchboxes at the base increased by one in each arrangement. However, despite recognizing this pattern, they made an incorrect calculation error and arrived at an incorrect answer.

The participant mentioned that their answer was wrong in response and acknowledged that they did not obtain 6 match boxes as expected. It appears they deviated from their intended approach or made a mistake during their calculations, leading to the incorrect answer. It was discovered that despite recognizing the pattern correctly, they encountered some difficulty or confusion in applying it accurately.

Participants were also asked, "How did you know that the 5^{th} arrangement of the match boxes in question 2B is 21?"
According to participants:

> *The first arrangement of the matchboxes is 3, the second arrangement is 6, and the third is 10. So, I got 21 because it is observed that the base is increased by one as it moves to the next arrangement of matchboxes on the shelf (R4). I counted the number of matchboxes and how they were laid from the first matchbox to the second and third. Counting how it is laid makes it easy to get the 21 at the fifth arrangement (R5).*

The participant's response suggests that they determined the value of the fifth arrangement of matchboxes (21) based on their observations of the previous arrangements. To calculate the value of the fifth arrangement, the participants applied a pattern they observed. They noticed that the value increased by one as they moved to the next arrangement of matchboxes on the shelf. To count the number of matchboxes in each arrangement, the participant likely looked at how the matchboxes were laid out from the first matchbox to the second and third ones. By counting the arrangement, they could determine that the fifth arrangement had a value of 21. Therefore, participants used the pattern of incrementing the base value by one for each subsequent arrangement and counted the matchboxes in a specific manner to arrive at the value of 21 for the fifth arrangement.

Furthermore, they were also asked, “How did you get your answer in question 2C?” this question asked, “Write a mathematical rule for the shopkeeper to use when arranging the matchboxes.” It is expected that participants would be able to form an exponential formula from the question. By just forming this formula, they score the full mark they deserve to score. However, this is what they have to say:

> *I did not understand question 2C, so I could not answer it. Though I tried doing it, I was not successful. At first, I used X to represent the base of the matchboxes, but later, I realized it was wrong and canceled it (R1). In solving question 2C, I have to develop a formula or rule, but how to state the formula is challenging for me. So, I could not do it (R2).*

Based on the participant's responses, they did not fully understand the question and could not answer it successfully. They mentioned not understanding the question and struggled to develop a formula or rule to solve it. Though some participants initially used the variable "X" To represent the base of the matchboxes, which is a good start, they later realized it was incorrect and had to ignore it. This suggests that they may have misunderstood the concept or requirements of the question. Besides, they acknowledge that stating the formula was challenging, indicating difficulty in expressing their thoughts or understanding the mathematical relationship within the pattern. This means they lack comprehension of the question, leading to their inability to provide the exponential formula or rule.

The D part is no exception after assessing participants' approaches to the A, B, and C of question 2. So, participants were also asked to narrate how they approached the question 2D. The researcher expected the participants to state a mathematical formula (see question 2D on APPENDIX C). However, this is what the participants had to say;

> *I tackled it, but later, it was kind of difficult for me. Though I do have some ideas, it's not enough, haha haha. I thought of the idea of mapping, and it was exponential, so it seems the arrangement of the tomatoes is in exponential order, but how to start was a problem (R3).*

It was observed that the participant mentioned that he found the task difficult and faced challenges in identifying the mathematical formula from the given pattern. He stated that he had some understanding but not enough to determine the solution confidently. The participant's mention of mapping suggests that he attempted to find a relationship or correspondence between elements in the pattern. He noticed a pattern that resembled exponential growth or distribution, where the arrangement of tomatoes appeared to follow an exponential order. However, the participant expressed uncertainty about how to start solving the problem. This suggests that he may have struggled to determine the initial conditions or find the specific mathematical expression that accurately described the pattern. Participant's difficulty in starting the problem suggests a lack of clarity regarding the initial conditions or the specific mathematical expression required.

Discussion

The finding from Newman's Error Analysis of prospective mathematics teachers' errors in solving algebraic concepts reveals some important insights into the challenges faced by these teachers. According to the analysis, the two most prevalent errors among prospective mathematics teachers when solving algebraic concepts were transformation and encoding errors. Transformation errors refer to difficulties in manipulating mathematical expressions, while encoding errors involve inaccurately transferring mathematical information (Wijaya et al., 2014). These findings suggest that the teachers struggled the most with performing mathematical operations and accurately representing mathematical ideas. Other types of errors common among prospective teachers in computing algebraic concepts include comprehension, process skills, and reading errors, with the latter receiving the least attention or being the least error type committed.

The results aligned with the outcomes reported in a study conducted by Son and Fatimah, 2019. Their research revealed that a cohort of students with low academic performance consistently made mistakes across all the criteria outlined by Newman. They found that these low-performing students made errors in understanding and encoding the material. On the other hand, students classified as medium achievers exhibited errors primarily in the process skill and encoding stages. Lastly, students categorized as high achievers tended to make mistakes, specifically in encoding. That is to say, students who perform poorly commit all the five stated errors, whereas average students commit process skill and encoding errors, with the good students committing errors at the encoding stage. The results were also confirmed by the study of (Fitriani et al., 2018) on analyzing students' errors in mathematical problem-solving based on Newman's error analysis. Their finding revealed that students made five types of errors in solving the problem of the derivative of an algebraic function: comprehension error, transformation error, process skill error, encoding error, and carelessness.

Though the study used Newman's error analysis to identify some errors committed by prospective mathematics teachers, there are several other error types that students and teachers commit when solving similar questions. For instance, Awuah (2018) posits that errors can be systematic and non-systematic—a systematic, deliberate error results from repeatedly providing incorrect information. Repeatedly using incorrect information throughout the process may result in correct answers, matching the incorrect information, making it extremely difficult to recognize systematic errors over time. Due to this, unless students are helped to become aware of them, these mistakes become persistent and are difficult for them to fix on their own. Inaccurate responses result from non-systematic errors, which are inadvertent and non-repetitive errors that happen frequently in calculations (Awuah, 2018). These errors are typically the consequence of carelessness and can be readily fixed by the students. Considering the concept established by Awuah, the type of error committed by the prospective mathematics teachers in this study can be categorized as systematic. These errors become permanent and cannot be easily corrected by the students themselves unless they are assisted to become aware of them.

In another study conducted by Marpa (2019), common errors in algebraic expressions are classified as errors committed in the classification of polynomials according to the number of terms, errors committed in the classification of polynomials according to the degree, errors committed in addition and subtraction of polynomials, error committed in translation, error committed in the multiplication of polynomials and error committed in the division of polynomials. These errors are also linked to the type of error stated by Newman and used in this study. Research has shown that most students made mistakes in computational errors in similar terms; addition and subtraction of polynomials accounted for 17.86%, 10.88%, and 12.04%, respectively (Thayarat et al., 2024). They further said careless errors were the biggest mistake in the multiplication of polynomials, accounting for 14.44%. The result shows that most students have computational errors but not conceptual ones. Hence, the errors (comprehension and transformation) committed by the prospective mathematics teachers align with the study conducted by Marpa (2019), which found the errors committed in translation under polynomials. The reasons for errors and methods to lessen them should be the subject of further research, particularly emphasizing the common errors identified in this study.

Conclusion

Newman's Error Analysis of prospective mathematics teachers' errors in selected schools indicates that the most prevalent types of errors were transformation errors (75.6%), comprehension errors (59.6%), and encoding errors (55.2%). This suggests that the prospective teachers struggled the most with manipulating mathematical expressions and understanding them after reading them and transferring mathematical information accurately. Process skills errors (39.2%) also contributed to the overall error profile, highlighting areas where the teachers faced challenges in understanding algebraic concepts, applying problem-solving strategies, and accurately interpreting mathematical problems or questions.

The super item test and interview protocol were used as the data collection instruments to help collect primary data for the study. The findings showed that Preservice mathematics teachers are likelier to commit transformation and comprehension errors, which could influence their students' ability to solve algebraic tasks. These errors committed would affect the progress of mathematics education in general in Ghana, specifically the students they teach.

Recommendations

Given the previous, error analysis could be included in the curriculum for teacher preparation since it will help to minimize or completely eradicate learner errors. It will help teachers recognize learner errors and help pre-service teachers eliminate them. Again, the study recommended that educators organize interventions and professional development programs that address error types committed to solving algebraic concepts, specifically transformation and comprehension errors. These can be done by Fostering collaboration and sharing best practices, improving teaching techniques, and enhancing assessment skills. By addressing these areas of challenge, colleges of education programs can better equip prospective mathematics teachers with the necessary skills to effectively teach and support their students in mathematics.

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References

Abdul, H. A., Nur Liyana, Z. A., & Marlina, A. (2015). Analysis of students' errors in solving higher-order thinking skills (HOTS) problems for the topic of functions. *A Canadian Centre of Science and Education, Asian Social Sciences 11, 21.*

Aboagye, E., & Yawson, J. A. (2020). Teachers' Perception of the New Educational Curriculum in Ghana. *African Educational Research Journal*, *8*(1), 6–12.

Adjei, E. A., & Oppong, R. A. (2024). Exploring pre-service teachers algebraic thinking levels: A mixed method research. International Journal of Current Education Studies (IJCES), 3(1), 1–13. https://doi.org/10.5281/zenodo.12188822

Agbofa, F. J. K., Mensah, D. K. D., Opoku-Amankwaah, K., Kyeremeh, S., & Adu-Boahen, A. O. (2023). Teachers' concerns on implementing the standards-based curriculum in Ghana: A Case of New Juaben North Municipal. *Creative Education*, *14*(5), 1076-1093.

Akufo-Addo, N. A. D. (2017). *Statement by the president of the Republic, Nana Addo Dankwa Akufo-Addo, at the commencement of the Free Senior High School Policy*.https://www.adomonline.com/ghana-news/full-text-presidents-speech-launch-free-shs/

Ardıç, M. A. A. A., & İşleyen, T. (2018). The Effect of Mathematics Instruction through Computer Algebra Systems on the Academic Achievements of Secondary Education Students: Turkey Example. Journal of Education and E-Learning Research, 5(3). https://doi.org/10.20448/journal.509.2018.53.165.173

Awuah, F. K. (2018). *Grade 12 learners' Problem-solving skills in probability*. A thesis was submitted in fulfillment of the Doctor of Philosophy degree requirement at the University of South Africa.

Blanton, M., Stephens, A., Knuth, E., Gardiner, A., Isler, I., & Kim, J. S. (2015). The development of children's algebraic thinking: The impact of a comprehensive early algebra intervention in 3rd grade. *Journal for Research in Mathematics Education*, 46 (1), 39-87.

Cangelosi, R., Madrid, S., Cooper, S., Olson, J., & Hartter, B. (2013). The negative sign and exponential expressions: Unveiling students' persistent errors and misconceptions. *The Journal of Mathematical Behaviour*, *32*(1), 69–82.

Casey, D. P. (1978). Failing students: A strategy of error analysis. *Aspects of motivation*, pp. 295–306.

Clarkson, P. C. (1980). The Newman error analysis extensions. *Research in mathematics education in Australia*, pp. *1*, 11–22.

Cleland, J. A. (2017). The qualitative orientation in medical education research. Korean J Med Educ. [PMC free article] [PubMed]

Collis, K. F., Romberg, T. A., & Jurdak, M. E. (1986). A technique for assessing mathematical problem-solving ability. Journal for Research in Mathematics Education, 17(3), 206-221.

Darmayanti, R., Baiduri, B., & Sugianto, R. (2022). Learning Application Derivative Algebraic Functions: Ethnomathematical Studies and Digital Creator Books. Jurnal Cendekia: Jurnal Pendidikan Matematika, 06(02), 2212–2227.

Egodawatte, G., & Stoilescu, D. (2015). Grade 11 Students' Interconnected Use of Conceptual Knowledge, Procedural Skills, and Strategic Competence in Algebra: A Mixed Method Study of Error Analysis. *European journal of science and mathematics education*, *3*(3), 289–305.

Ernest, P. (2014). The knowledge, beliefs, and attitudes of the mathematics teacher: A model. *Journal of education for teaching, 15*(1), 13–33.

Fitriani, H. N., Turmudi, T., & Prabawanto, S. (2018). Analysis of students' errors in mathematical problem solving based on Newman's error analysis. In *International Conference on Mathematics and Science Education* (Vol. 3, No. 1, pp. 791-796).

Frenzel, A. C., Goetz, T., Pekrun, R., & Watt, H. M. (2010). Development of mathematics interest in adolescence: Influences of gender, family, and school context. Journal of adolescent research, *20*(2), 507-537.

Jupri, A., & Drijvers, P. (2016). Student Difficulties in Mathematizing Word Problems in Algebra.Eurasia Journal of Mathematics, Science & Technology Education, 12(9), 2481-2502.

Jupri, A., Drijvers, P., & van den Heuvel-Panhuizen, M. (2014). Difficulties in initial algebra learning in Indonesia. *Mathematics Education Research Journal*, *26*(4), 683–710.

Knuth, E., Stephens, A., Blanton, M., & Gardiner, A. (2016). Build an early foundation for algebra success. Phi Delta Kappan, 97(6), 65-68. https://doi.org/10.1177/0031721716636877

Malihatuddarojah, D., & Prahmana, R. C. I. (2019). Analisis Kesalahan Siswa Dalam Menyelesaikan Permasalahan Operasi Bentuk Aljabar. *Jurnal Pendidikan Matematika, 13*(1), 1–8.

Marpa, E. P. (2019). Common Errors in Algebraic Expressions: A Quantitative-Qualitative Analysis. *International Journal on Social and Education Sciences*, *1*(2), 63–72.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school* *mathematics*. Reston, VA: Author.

National Mathematics Advisory Panel. (2008). *The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.

Newman, M. A. (1977). An analysis of sixth-grade pupils' errors on written mathematical tasks. *Victorian Institute for Educational Research Bulletin*, pp. 39, 31–43.

Ojose, B. (2011). Mathematics literacy: can we put the mathematics we learn into everyday use? *Journal of Mathematics Education*, 4(1), 89–100.

Polya, G. (2004). *How to solve it: A new aspect of mathematical method* (246). Princeton University Press.

RAND Mathematics Study Panel. (2003). *Mathematical proficiency for all students: Toward a strategic research and development program in mathematics education*. Santa Monica, CA: RAND.

Sah, R. W. A., Laila, A. R. N., Setyawati, A., Darmayanti, R., & Nurmalitasari, D. (2023). Misconception Analysis of Minimum Competency Assessment (AKM) Numeration of High School Students from Field Dependent Cognitive Style. JEMS: Jurnal Edukasi Matematika Dan Sains, 11(1), 58–69. https://doi.org/10.25273/jems.v11i1.14112

Sari, D. P., Fiantika, F. R., & Pd, M. (2018). Students' Algebraic Thinking Processes in Mathematics Problem Solving at Low Mathematic Ability Students Based on Quantitative Reasoning Ability. Edumatika Jurnal Riset Pendidikan Matematika, 1(November), 29–35.

Saundarajan, K., Osman, S., Kumar, J. A., Daud, M. F., Abu, M. S., & Pairan, M. R. (2020). Learning Algebra using Augmented Reality: A Preliminary Investigation on the Application of Photomath for Lower Secondary Education. International Journal of Emerging Technologies in Learning (IJET), 15(16). https://doi.org/10.3991/ijet.v15i16.10540

Sekaryanti, R., Cholily, Y. M., Darmayanti, R., Rahma, K., Prasetyo, B., & Maryanto, A. (2022). Analysis of Written Mathematics Communication Skills in Solving Solo Taxonomy Assisted Problems. Jurnal Edukasi Matematika Dan Sains, 10(2), 395–403. https://doi.org/10.25273/jems.v10i2.13707

Sikukumwa. (2017). Types of Strategies Used to Solve Algebraic Word Problems by Grade 12 Ordinary Level Mathematics Learners of Kavango East Region of Namibia, (April).

Sinaga, N. Y., Siagian, M. V, & Hasibuan, A. M. (2021). Kesulitan Siswa SMA Dalam Menyelesaikan Soal Cerita Matematika Materi Peluang. ALGEBRA: Journal of Mathematics ..., 1(1)

Son, A. L., & Fatimah, S. (2019, October). An analysis of student error in algebraic problem solving based on Polya and Newman theory. In *Journal of Physics: Conference Series*, *1315*(1), 12069. IOP Publishing.

Star, J. R., Foegen, A., Larson, M. R., McCallum, W. G., Porath, J., Zbiek, R. M., Caronongan, P., Furgeson, J., Keating, B., & Lyskawa, J. (2015). *Teaching strategies for improving algebra knowledge in middle and high school students.* Educator's Practice Guide. What Works Clearinghouse. ™ NCEE 2015-4010. What Works Clearinghouse.

Sulistiawati, & Surgandini, A. (2019). Investigating the learning obstacles and the self-confidence of students in college in terms of material understanding ability in linear algebra courses. International Journal of Scientific and Technology Research, 8(9).

Thayarat, E., Parinya Sa, N., Mingkhuan, P., Ratchanikorn, C. (2024). Identifying common errors in polynomials of eighth-grade students. *International Journal of Evaluation and Research in Education (IJERE),*13(1) 57-68. DOI: 10.11591/ijere. v13i1.25131

Vamsi Sagar, P., & Phani Krishna Kishore, M. (2019). Connections between graphs and sheaves. Journal of Algebra and Related Topics, 7(2). https://doi.org/10.22124/jart.2020.13887.1154

Wijaya, A., van den Heuvel-Panhuizen, M., Doorman, M., & Robitzsch, A. (2014). Difficulties in solving context-based PISA mathematics tasks: An analysis of students' errors. *The Mathematics Enthusiast*, *11*(3), 555-584.

Wilson, M., & Iventosch, L. (1988). The partial credit model was used to investigate responses to the structured subtest. Applied Measurement in Education, 1(4), 319-334.

Wood, Bruner, D. J., & Ross, G. (1976). The role of tutoring in problem-solving. Journal of Child Psychology and Child Psychiatry, 17, 89–100.

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