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Mathematical Critical Thinking Skills: Students' Analytical Ability on the Topic of Circles

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Abstract

Critical thinking skills are one of the important competencies of the 21st century that need to be developed through mathematics learning, especially in flat geometry, namely circles. One important indicator of critical thinking is analytical skills, namely the ability to analyze information, identify relationships between concepts, and draw rational conclusions. This study aims to describe students' mathematical critical thinking skills in the indicator of analysis in circles. The research used a descriptive quantitative approach with a sample of 31 eighth-grade students at a public junior high school in Padang City. The instrument consisted of one essay question that asked students to analyze two different arguments related to the concepts of diameter and chord, choose the correct argument, and provide logical reasons based on the concept of circles. Scoring was done using an analytical rubric with a score range of 0–5. The results showed that only 3.22% of students were able to analyze correctly with logical reasoning, 6.45% of students could choose the correct argument but without a conceptual explanation, 80.65% of students were unable to analyze correctly and tended to copy the arguments in the questions, and 9.68% of students did not provide answers. These findings indicate that students' mathematical critical thinking skills, particularly their ability to analyze circle-related material, are still low. This study emphasizes the importance of mastering basic concepts as a foundation for promoting critical analysis skills in mathematical problem solving.

Keywords: Analysis; Circles; Critical thinking skills; Mathematics; 21st Century Skills.

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1. Introduction

The 21st century requires students to possess 4C skills as preparation for facing developments in science, technology, and global life challenges. The 4C skills are critical thinking, creativity, communication, and collaboration (Azkiana et al., 2025; Jelodari et al., 2025; Vithanage & Nakashima, 2025). Among these four skills, critical thinking is critical. Critical thinking is widely recognised as a vital 21st-century skill, essential for navigating the complexities of modern life and work (Afiyanti et al., 2025; Pogrebnaya & Mikhailova, 2023). This skill is also a higher-order thinking skill that helps individuals evaluate information, consider various perspectives, and make logical decisions (Payadnya et al., 2023; Rahmawati et al., 2024). Critical thinking involves analysing and evaluating information using evidence, observation, and logical arguments to conclude (Berg et al., 2023; Rupe & Voas, 2025).

Critical thinking skills are one of the key competencies that must be developed in education, primarily through mathematics learning. This skill is considered an important learning outcome in school and higher education (Kania et al., 2024; Vincent-Lancrin, 2024). In the professional world, critical thinking skills are necessary for problem-solving, decision-making, and adaptation to technological advances and globalisation (Horton et al., 2017). These skills involve analysing, evaluating, and synthesising information to produce appropriate decisions and practical solutions (Madden & Dedic, 2022; Pogrebnaya & Mikhailova, 2023; Saikia & Roy, 2024; Trúsiková & Velmovská, 2022). In addition to cognitive aspects, critical thinking also involves affective aspects, such as self-regulation (Utami, Probosari, et al., 2019), which enables individuals to deal with complex problems, make data-based decisions, and adapt to rapid social, technological, and environmental changes (Abbas & Rozimela, 2025; Saikia & Roy, 2024).

Mathematics is a subject closely related to logical reasoning (Su et al., 2016), analysis (Ali et al., 2018), and problem-solving (Jawad, 2022; Kureethara & Joseph, 2017). Mathematics learning fosters critical thinking by encouraging students to construct logical chains, detect faulty arguments, and manipulate concepts appropriately (Abayeva et al., 2024; Georgieva & Nikolova, 2019). Therefore, critical thinking is important in understanding concepts, connecting ideas, and solving mathematical problems. One prominent indicator is analytical ability, which involves breaking down information, identifying relationships between concepts, and drawing rational conclusions. Critical thinking is analytical and focuses on evaluating information to make reasoned judgments (Hidayati et al., 2019; Sun & Hui, 2006; Zhang et al., 2023). Mathematical analysis supports the development of problem-solving skills and more fundamental mathematical thinking (Zhang, 2021). It fosters a systematic mindset by breaking down complex problems into manageable parts (Hariri & Kania, 2025; Levin & Verner, 2020).

The topic of circles in eighth-grade junior high school requires good analytical skills because it covers concepts, properties, and applications in everyday life. Analytical skills include three things: first, breaking down information into smaller parts to improve understanding (Burhanuddin et al., 2019; Castle, 2010; Hitchcock, 2017); second, identifying relationships between concepts to form a coherent understanding (Burhanuddin et al., 2019; Utami, Saputro, et al., 2019; Waluyo, 2021); and third, drawing rational conclusions based on evidence (Burhanuddin et al., 2019; Hitchcock, 2017; Wulandari & Wutsqa, 2019). This analysis is at the core of mathematical critical thinking skills, which combine information evaluation, problem solving, and conceptual understanding (Susanti et al., 2020; Wibawa et al., 2025).

Critical thinking skills are closely related to conceptual understanding. Ramlo (2019) found that students with higher levels of critical thinking had better conceptual understanding in physics. Similarly, Kriswanto et al. (2020) showed that integrating GeoGebra in geometry learning improved students' ability to make assumptions, predictions, and hypotheses, thereby developing their critical thinking skills. Previous studies have also shown that conceptual understanding is the foundation of problem solving because every mathematical concept is built on prior knowledge (Al-Mutawah et al., 2019; Cahyani et al., 2024; Ningrum et al., 2022). Procedural understanding is important, but it is conceptual understanding that enables students to know why a procedure works and when to apply it (Chueh & Zhao, 2025; Khairunnisa & Darhim, 2019; Kilp-Kabel & Mädamürk, 2024; Rittle-Johnson, 2017). With a strong conceptual understanding, students can think logically and analytically and connect various mathematical ideas (Chimmalee & Anupan, 2022; Kholid et al., 2021; Prayitno et al., 2018).

Research by Abakah and Brijlall (2024) shows that interactive didactic assessment based on APOS theory can improve students' metacognition and critical thinking skills in circle-related material. However, the research focuses more on complex non-routine questions, so it does not fully reveal students' analytical abilities regarding basic circle concepts. In fact, analysis of basic concepts is the foundation of mathematical critical thinking and a determinant of students' conceptual understanding. Conceptual understanding is crucial because it is the basis for problem solving; every mathematical concept is built on prior knowledge (Al-Mutawah et al., 2019; Cahyani et al., 2024; Ningrum et al., 2022). Thus, research on students' critical thinking skills, especially on the analysis indicator, should begin with ensuring mastery of basic concepts. This requires the design of assessment instruments that contain complex or non-routine questions and questions that measure basic conceptual understanding.

Furthermore, research by Kriswanto et al. (2020) found that integrating GeoGebra into geometry learning effectively improves critical thinking skills through the ability to make assumptions, predictions, and hypotheses. However, this study focused more on technological support that facilitates creative thinking rather than on in-depth exploration of students' analytical skills regarding the relationship between circle concepts. This leaves room for further research to examine students' analytical abilities, without relying entirely on technology-based media.

The research by Putri and Khadijatuzzahra (2025) focused on students' mathematical creative thinking skills on spatial figures. The results showed that the elaboration indicator was still lower than flexibility, originality, and fluency. Although this study contributed to the study of higher-order thinking skills, its focus was more on creativity and did not touch on critical thinking skills, especially analytical skills. Given the close relationship between critical and creative thinking, this gap is important to follow up on. First, research shows a positive correlation; an increase follows in critical and creative thinking scores (Arsih et al., 2023; Hidayati et al., 2019; Rosba et al., 2021). Second, critical thinking skills can predict creative thinking, where a one-point increase in critical thinking scores can increase creative thinking scores by 0.913 points (Arsih et al., 2023). Third, critical and creative thinking are closely related and work together in problem solving, where critical thinking helps evaluate and refine creative ideas (Koçoğlu & Kanadlı, 2025; Lau, 2011; Mittal, 2021). Although different, they are complementary: critical thinking provides a framework for evaluating ideas, while creative thinking brings new perspectives that enrich critical analysis (Álvarez-Huerta et al., 2022; Feist, 2024; Hidayati et al., 2019; Sun & Hui, 2006; Zhang et al., 2023). Thus, from these four facts, it can be concluded that critical thinking skills, particularly analytical skills, can be seen as the foundation for creative thinking skills in mathematical problem solving.

Finally, research by Kania et al. (2025) produced a valid test instrument for junior high school students' higher-order thinking skills (HOTS) in mathematics. This instrument contains questions that measure analytical skills. However, it emphasizes the ability to break down complex problems into simpler parts rather than the critical evaluation of information, which is the core of analysis (Saputro & Sunarno, 2021). In fact, the core of analytical skills in critical thinking is to evaluate information to make appropriate and reasoned decisions critically (Hidayati et al., 2019; Sun & Hui, 2006; Zhang et al., 2023). Thus, there is still an opportunity for research to develop an understanding of student analysis, not only in breaking down problems, but also in evaluating information and drawing logical conclusions on circle material.

Based on the description of previous research, critical thinking skills, particularly analytical skills, have not been explored in depth in mathematics learning. Abakah and Brijlall (2024) emphasize assessing complex non-routine problems but neglect analysing basic circle concepts. Kriswanto et al. (2020) highlight the effectiveness of GeoGebra in improving critical thinking skills, but focus more on technology-based creativity than pure analysis of geometric concepts. Putri and Khadijatuzzahra (2025) examine aspects of creative thinking, but do not touch on analytical skills, which are at the core of critical thinking. Meanwhile, Kania et al. (2025) have developed a valid HOTS instrument, but the analytical aspects explored are limited to solving complex problems rather than evaluating information and drawing logical conclusions. These research gaps indicate the need for a specific study on mathematical critical thinking skills focusing on students' analytical abilities on circles, which is fundamental and applicable in everyday life and other disciplines. The urgency of this research is even greater, considering that analysis is at the core of critical thinking skills and is closely related to creative thinking skills (Álvarez-Huerta et al., 2022; Arsih et al., 2023; Feist, 2024; Hidayati et al., 2019; Koçoğlu & Kanadlı, 2025; Lau, 2011; Mittal, 2021; Rosba et al., 2021; Zhang et al., 2023).

The urgency of this study lies in the importance of revealing students' mathematical critical thinking skills, particularly their analytical abilities, on the topic of circles. Previous studies have examined critical and creative thinking skills, but studies specifically examining students' analytical abilities in circles are still limited. In fact, circles, as fundamental flat shapes, play an important role in science and engineering (Miller et al., 2011; Vainsencher & Bruckstein, 2008) as well as in design and architecture applications (Al-rabtah & Al-Banawi, 2023; Marjanović, 2007).

Therefore, this research is urgently needed to provide a clearer picture of students' mastery of mathematical critical thinking skills, particularly analytical skills, on the topic of circles. This research's novelty lies in its specific focus on analytical skills as a core indicator of mathematical critical thinking in the context of circles in the 8th grade of a public junior high school.

Previous studies have emphasised the general aspects of critical thinking or creative thinking skills, while this study highlights the role of conceptual analysis in understanding and solving circle problems. Thus, this study expands the literature on mathematical critical thinking skills by in-depth studying one of its leading indicators.

Theoretically, the results of this study can enrich the study of the relationship between critical thinking skills, conceptual understanding, and mathematical problem solving, particularly in circle geometry. Practically, this study provides input for teachers in designing learning strategies that encourage students to develop analytical skills and for assessment developers in designing instruments that are more balanced between routine, non-routine, and fundamental concept evaluation problems. Thus, this study provides academic contributions and tangible benefits in improving the quality of mathematics learning in schools.

2. Methods

This study used a descriptive quantitative approach to describe students' mathematical critical thinking skills, particularly their analytical abilities on circles. The research population consisted of all 255 eighth-grade students from eight classes at a public junior high school in Padang City. The research sample was determined using purposive sampling based on the criterion that students had learned the same material on circles. Based on these criteria, one class containing 31 students was selected as the research sample.

The research instrument consisted of a written test question to measure mathematical critical thinking skills in the analysis indicator. The question required students to read information from statements from two characters (Ani and Budi) debating the concepts of chord and diameter in a circle. Students were asked to analyse the information, decide which argument was correct, and provide logical reasons based on the mathematical concept of circles. Thus, this question emphasised correct or incorrect answers and students' ability to reason, identify the suitability of information, and present supporting mathematical reasons. Experts in the field of mathematics education have validated this question.

Student answers are scored using an analytical rubric with a score range of 0–5. The score is divided into two parts, namely: (1) accuracy in choosing the correct information, with a score of 0 if incorrect and one if correct, and (2) accuracy and logic of the reasoning in choosing the information, with a score of 0 if incorrect and four if correct. Thus, the minimum score is zero and the maximum score is 5.

Indicator : **Analyze (analysis)**
(recognize whether a statement is an evidence-based argument or merely an opinion/information without evidence).

Question:

Ani and Budi are two friends who enjoy discussing mathematics. One day, while sitting in the school garden, they debated the concept of a chord in a circle.

Ani said, **"The diameter is also a chord in a circle."** However, Budi disagreed. He replied, **"A chord does not pass through the circle's center, while the diameter does. The diameter is special because it divides the circle into two equal parts. Therefore, the diameter is not a chord."** They became unsure and finally decided to ask their teacher.

Question:

In your opinion, whose argument is correct? Explain your answer using logical reasoning and based on mathematical concepts about circles!

Answer:

Ani's argument is correct. (score 1)

Reasons:

1. 1. A chord is a line segment connecting two points on a circle (circle arc). Since a diameter also connects two points on a circle, and even though a diameter passes through the center of the circle, a diameter is still a chord. (score 4), or
2. 2. A diameter is the longest chord. (score 4)

Figure 1 Critical Thinking Skills Test Questions on Analytical Ability Indicators

3. Results and Discussion

Based on the analysis of students’ written responses, four distinct categories of responses were identified. First, students who were able to analyse the problem accurately by selecting the correct argument and supporting their choice with logical reasoning grounded in relevant mathematical concepts. Second, students who were able to identify the correct argument but provided insufficient justification, as their explanations merely restated or quoted parts of the question without demonstrating deeper understanding. Third, students who failed to identify the correct argument and whose explanations were limited to reproducing information from the question indicated a lack of analytical engagement. Finally, students who did not provide any response at all. These categories reflect varying levels of cognitive engagement and highlight the differences in students’ reasoning processes when confronted with argumentation-based mathematical tasks.

3.1.1. Able to analyse by selecting which argument is correct and providing logical reasoning based on mathematical concepts

The correct answer is Ani's argument. The logical reasoning is: (1) The concept of a chord is a line segment connecting two points on a circle (circle arc). Because a diameter also connects two points on a circle, and even though it passes through the centre of the circle, it is still a chord; or (2) A diameter is the longest. Both reasons are correct, and students are considered correct if they give at least one of these reasons. Here is an excerpt from one of the students' answers:

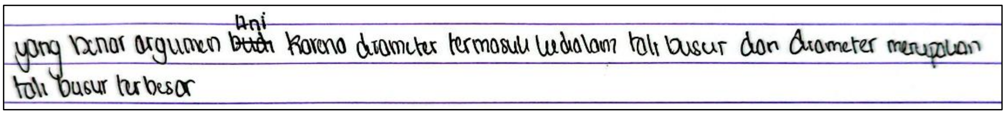
yang benar argumen ~~Budi~~ Ani karena diameter termasuk kedalam tali busur dan diameter merupakan tali busur terbesar

Figure 2 One of the student's answers is correct with a logical explanation (Indonesian version)

Ani's answer is correct because the diameter is included in the bowstring, and the diameter is the largest.

Figure 3 One of the student's answers is correct with a logical explanation (English version)

Students who gave answers like those in Figures 2 and 3 got the maximum score of 5. One student (3.22%) fell into this type 1 category, where students could choose the correct argument accompanied by logical reasoning based on the concept of circles. This shows that students were able to analyse well. However, based on the percentage, very few or almost no students have critical thinking skills in the indicator of information analysis ability.

3.1.2. Able to analyse by choosing whose argument is correct, but the reasons given only quote the question

This type of response is considered partially correct in terms of identifying the appropriate answer. However, the reasoning provided does not align with the targeted mathematical concept of the circle, as it merely reproduces information stated in the problem, specifically the argument presented by "Ani." Such reasoning reflects a superficial level of understanding, as students rely on quoting the given statement rather than demonstrating independent analytical thinking or applying relevant conceptual knowledge. An excerpt from one of the students' responses is presented below:

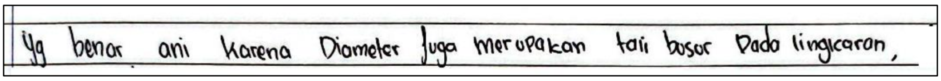
Yg benar ani karena Diameter juga merupakan tali busor pada lingkaran,

Figure 4 One of the student's answers is correct, but the reasoning is incorrect (Indonesian version)

The correct answer is Ani's, because the diameter is also the chord of a circle.

Figure 5 One of the student's answers is correct, but the reasoning is not appropriate (English version)

Two students (6.45%) fell into this second type. Students who gave answers such as those shown in Figures 4 and 5 received a score of 1 because they could only choose whose argument was correct, but their reasoning did not match the mathematical concept in question. The students only quoted the correct argument, namely Ani's statement, "The diameter is also a chord of a circle." In fact, the students should have explained why they agreed that the diameter is also a circle chord, as in the previous type 1 answer.

3.1.3. Unable to analyze by choosing which argument is correct and only quoting the question as the reason

This third type of answer proves that students cannot analyze the correct argument. In addition, students only quote back the argument they consider correct (even though it is wrong). Here is an excerpt from one of the students' answers:

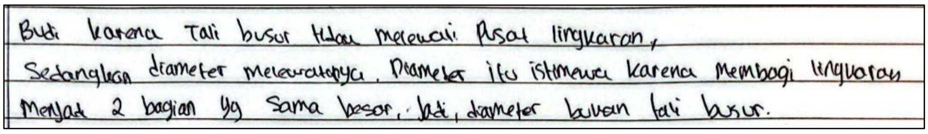
Budi karena Tali busur tidak melewati Pusat lingkaran,
Sedangkan diameter melewatinya. Diameter itu istimewa karena membagi lingkaran
Menjadi 2 bagian yg Sama besar, Jadi, diameter bukan tali busur.

Figure 6 One example of an incorrect student answer (Indonesian version)

> Budi, because the bowstring does not pass through the circle's center, whereas the diameter does. The diameter is special because it divides the circle into two equal parts. So the diameter is not a bowstring.

Figure 7 One example of an incorrect student response (English version)

Students who gave answers like those in Figures 6 and 7 received a minimum score of 0. Twenty-five students (80.65%) fell into this third type, where students were unable to analyze the correct argument. Although they made an effort to answer, this still showed that they were unable to analyze the correct answer.

3.1.4. No answer

This fourth type of response proves that students cannot analyze which argument is correct by not providing any answer. Students who did not provide an answer received a minimum score of 0. There were 3 students (9.68%) who fell into this fourth type, where students did not answer the question.

The results of the study show that the majority of students, namely 80.65%, are unable to analyze information correctly and tend to only repeat the arguments from the questions without providing logical reasons. In fact, 9.68% of students did not provide any answers at all. Meanwhile, only 3.22% of students were able to choose the correct argument and provide logical reasons in accordance with the concept of circles, and 6.45% of students were able to choose the correct argument but were unable to relate it to a conceptual explanation. These findings confirm that students' mathematical critical thinking skills in the analysis indicator are still very low, especially in relating the information in the question to the basic concept of circles.

This finding aligns with that of Wicaksono and Martyanti (2019), who found that the most common mistake made by eighth-grade students at a junior high school in Purworejo was drawing incorrect conclusions. This includes drawing conclusions about which arguments are correct. We suspect that the findings of 80.65% of students being unable to analyze information correctly and tending to simply repeat the arguments from the questions without providing logical reasons, as well as 9.68% of students not providing any answers at all, are due to the students' low level of understanding of the concept of circles. This is also in line with the findings of Wicaksono and Martyanti (2019), which state that student errors on the topic of circles are conceptual understanding errors of 16.93%.

Our following assumption is that it is possible that students actually understand the concepts related to diameter and chord in circles. However, students find it difficult to analyze when faced with a new contextual problem in the form of a story question. The concepts understood by students are still not deep enough. This is in line with the findings of Michael (2022) that students have difficulty transferring knowledge from one topic to another, which hinders their ability to apply conceptual understanding to new problems.

Our assumption is not without reason. This has been proven by Kartono et al. (2019) that between the aspects of mathematical critical thinking and theoretical metacognition, when observed in depth, there is a close relationship. Kartono et al. (2019) found that when someone is about to conclude a statement or assess the validity of an argument, they need prior knowledge and strategies to conclude that all of these are aspects of metacognition. Therefore, our assumption explaining why almost all students cannot analyze the correct argument regarding diameter and chord is that their conceptual understanding is still low, which is the most likely cause.

Students who cannot analyze arguments correctly can be considered students with low critical thinking skills. This is supported by the findings of Putri et al. (2022) that students with high abilities can interpret questions and make analyses. However, there are still students who make mistakes in their calculations. Students in the middle category have been able to interpret and analyze questions. However, their inferences and evaluations are still incomplete. At the same time, students in the low category have not been able to interpret, analyze, or evaluate questions to conclude a mathematical question.

A unique thing that happened in the students' answers was that they were able to analyze which arguments were correct but were unable to provide logical reasons for their answers because they only copied the information in the questions or the information that became the arguments. In this case, we assume students cannot complete the questions. This is in line with the findings of Palazzo et al. (2010) that students are often forced to copy due to time constraints, especially when they start their homework late and face an approaching deadline. This may also occur when students work on test questions (not just homework). The next possible cause is difficulty with the material. Although the material on the concepts of diameter and chord is not particularly complex, it is possible that students do not yet understand the concepts, or tend only to memorize the elements of a circle, so that when faced with new problems, students fail to analyze them and end up copying information from the question rather than providing an answer. This is in line with the findings of Buagayan et al. (2024) that difficulties in understanding mathematical concepts and self-regulation can encourage students to copy answers as a coping mechanism. However, based on the findings of Moore (2013) and Tosuncuoglu (2018), the solution to overcome students' low critical thinking skills, especially in the analysis ability indicator, is to use discussions in classroom learning to deepen understanding and analysis of arguments.

4. Conclusions

This study reveals that students' mathematical critical thinking skills, particularly their analytical abilities in the topic of circles, are still at a low level. These findings emphasize that understanding basic concepts is an important prerequisite for students to evaluate information, select correct arguments, and provide logical reasons in solving mathematical problems. Theoretically, the results of this study expand the study of the relationship between concept mastery, critical thinking skills, and mathematical problem solving, especially in the realm of circle geometry. Practically, this study provides input for teachers to design learning strategies that emphasize basic conceptual understanding while encouraging critical analysis through contextual questions and class discussions, as well as for assessment developers to design instruments that balance routine and non-routine questions and information evaluation.

However, this study has limitations regarding the relatively small sample size, the analysis of the causes of low critical thinking skills that have not been empirically validated, and the focus on only one analysis indicator. Therefore, further research is recommended to involve a larger sample, use data triangulation such as interviews or questionnaires, and examine other critical thinking skill indicators such as deduction, interpretation, inference, evaluation, and explanation.

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Conflict of Interest

The authors declare that there are no conflicts of interest.

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