INTERNATIONAL JOURNAL OF GEOMETRY RESEARCH AND INVENTIONS IN EDUCATION
Vol. 1 No. 2 (2024) 58-65

GRADIENT

https://journals.eduped.org/index.php/gradient
E-ISSN 3036-959X

Analysis of Student Errors in Solving Problems on Three-Dimensional Shapes with Flat Surfaces: A Qualitative Study on Eighth-Grade Students

Wiwid Febriyanti[1], Wahyuddin[2*], Mutmainnah[3]

[123]Universitas Muhammadiyah Makassar, Jalan ST Alauddini Nomor 259 Makassar, 90221, Indonesia

*Corresponding author: wahyu@unismuh.ac.id

Article Info

Received Oct 21, 2024
Revised Nov 10, 2024
Accepted Dec 12, 2024

Abstract

This study analyzes errors made by eighth-grade students in solving problems involving three-dimensional geometric shapes with flat surfaces and examines the factors causing these errors. Using a qualitative descriptive approach, three students with varying error levels (high, medium, low) were selected for analysis through written tests and in-depth interviews. Data were processed using the Miles and Huberman model: data reduction, data presentation, and conclusion drawing. Findings identified four main error types: factual, conceptual, principle, and operational. High-error students mainly struggled with conceptual understanding and operations, while low-error students primarily made operational errors. The results highlight that weak conceptual understanding and cognitive overload were the key factors in these errors. This study implies that teaching strategies should enhance both conceptual and procedural knowledge, utilizing interactive and manipulative learning methods to reduce cognitive load and improve student performance in geometry.

Keywords: Conceptual understanding; Cognitive load; Geometry; Student errors; Three-dimensional geometric shapes.

How to cite: Febriyanti, Wahyuddin, and Mutmainnah. (2024). Analysis of Student Errors in Solving Problems on Three-Dimensional Shapes with Flat Surfaces: A Qualitative Study on Eighth-Grade Students. *Journal of Geometry Research and Innovation in Education*, 01(02), 58-66, doi. https://doi.org/10.56855/gradient.v1i2.1198

1. Introduction

Mathematics education in junior high school (SMP) is a crucial foundation in developing students' logical, analytical, and critical thinking skills. One of the key topics in the mathematics curriculum is three-dimensional shapes with flat surfaces, which include cubes, rectangular prisms, prisms, and pyramids. A deep understanding of these shapes is essential for developing spatial visualization skills, geometric comprehension, and the application of mathematics in real-world contexts. Despite the fact that this material has been taught since elementary school, many junior high school students still struggle to fully grasp the concepts of three-dimensional shapes. This is supported by various studies that indicate many students tend to make errors when solving problems related to three-dimensional shapes with flat surfaces (Wijaya et al., 2019).

In the context of mathematics education, exploring and understanding the sources of students' errors is essential, as these errors can indicate conceptual weaknesses or difficulties in applying mathematical procedures. These errors not only occur due to a lack of understanding of concepts but can also be caused by psychological factors such as mathematics anxiety, improper learning strategies, or even poor quality of instruction in the classroom (Litke, 2019). Therefore, analyzing student errors in solving problems related to three-dimensional shapes with flat surfaces is crucial, both from a pedagogical and theoretical perspective, to improve the quality of mathematics instruction in schools.

The relevance of this study is also grounded in the global challenges of mathematical literacy faced by many countries, including Indonesia. According to the Programme for International Student Assessment (PISA) report, Indonesian students' mathematics performance is below the international average, particularly in problem-solving and the application of mathematical concepts in everyday life (OECD, 2018). This low performance underscores the importance of research that focuses on student errors in understanding and applying mathematical concepts, particularly in geometry topics such as three-dimensional shapes with flat surfaces. A better understanding of the types of errors students frequently make and their causes can significantly contribute to improving teaching strategies and developing more effective assessment methods.

Research on student errors in mathematics has gained widespread attention in recent decades. Various theories about errors in learning mathematics have been developed, one of which is the Van Hiele theory on the development of geometric understanding. According to this theory, students' understanding of geometry progresses through several stages, from visual recognition to logical deduction (Van Hiele, 1986). However, research shows that many junior high school students do not reach an adequate level of understanding, leading them to make errors in solving complex geometry problems (Mason, 2018).

Another study by Haser and Star (2020) indicates that students' errors in solving problems related to three-dimensional shapes are often linked to difficulties in connecting visual representations with symbolic or mathematical representations. For example, students often misunderstand the concepts of surface area and volume due to their inability to accurately visualize three-dimensional shapes. Additionally, Litke (2019) found that the use of conventional teaching models, which overemphasize memorizing formulas without conceptual understanding, can exacerbate the situation, where students focus more on solving problems mechanically rather than on gaining a deep understanding of the concepts.

This condition is reinforced by the findings of Wijaya et al. (2019), which indicate that students' difficulties in solving problems related to three-dimensional shapes with flat surfaces are also caused by low higher-order thinking skills, which are necessary for understanding geometric concepts deeply. The study suggests that learning interventions should emphasize visual and manipulative approaches to help students overcome these difficulties.

Although many studies have examined student errors in mathematics, particularly in geometry, there are still some gaps that need to be addressed. Most previous studies have focused more on identifying the types of errors students generally make, but fewer have explored the specific causes of these errors, especially in the context of three-dimensional shapes with flat surfaces at the junior high school level. Moreover, the approaches used in previous studies have often been quantitative, without providing deep insights into the students' thought processes when they make errors.

This study aims to fill these gaps by using a more in-depth qualitative approach to identify the types of errors and the factors that influence them. Thus, this study not only seeks to identify errors but also to understand how students think and process information when solving problems related to three-dimensional shapes with flat surfaces. This approach is expected to provide more comprehensive insights into how to improve geometry learning processes in schools.

Based on the above explanation, the main problem to be investigated in this study is as follows: "What types of errors do eighth-grade students make when solving problems related to three-dimensional shapes with flat surfaces, and what are the factors that cause these errors?"

This research is expected to make significant contributions both theoretically and practically. Theoretically, this study will enrich the academic literature on student errors in mathematics education, particularly in the context of geometry and three-dimensional shapes. By using a qualitative approach, this study will also provide new insights into how students think and process geometric concepts, which have not been extensively explored in previous research. Practically, the findings of this study can serve as a basis for designing more effective teaching strategies in teaching three-dimensional shapes with flat surfaces. Teachers can use these findings to identify and correct common conceptual errors made by students, and to develop teaching methods that focus more on conceptual understanding rather than mere formula memorization. Additionally, this study has important implications for the development of the mathematics curriculum at the junior high school level, emphasizing the importance of using visual and manipulative approaches in geometry instruction.

2. Methods

This study employs a descriptive qualitative design with the aim of analyzing students' errors in solving problems related to three-dimensional shapes with flat surfaces in the eighth grade of MTs Muhammadiyah Tallo as a private islamic school in the city of Tallo. A descriptive design was chosen because it is suitable for providing a detailed depiction of phenomena, while the qualitative approach allows for in-depth exploration of students' errors and the factors causing them. The research subjects were selected purposively, consisting of three students who made errors categorized as high, medium, and low. This selection was based on the results of a written test, as well as the students' ability to communicate effectively during interviews.

The written test was used as the primary instrument to identify the types of errors made by students, such as factual, conceptual, principled, and operational errors. Unstructured interviews were then conducted to further explore the causes of students' errors, with interview guidelines developed after the test data had been collected. Data collection was carried out in two stages: a written test administered to all students in class VIII-2 , and in-depth interviews with the three selected students. It involved 16 students with 10 female and 6 male students.

Data analysis followed the Miles and Huberman model, which involves data reduction, data presentation, and conclusion drawing Asipi, Rosalina, & Nopiyadi. (2022). The reduced data was presented in a narrative form to facilitate interpretation. The validity of the data was tested using triangulation techniques, which involved cross-checking data from various sources and methods. In terms of ethics, this research received permission from the school, and informed consent was obtained from the participants. Their privacy and confidentiality were maintained throughout the study.

However, this research has limitations, particularly related to the small number of subjects—only three students—which may affect the generalizability of the findings. Additionally, the unstructured interviews carry the potential for interpretative bias, but triangulation efforts were made to minimize the impact of this bias. The chosen research design and methods were selected because they are well-suited to answering the research questions related to the types and causes of students' errors in solving problems involving three-dimensional shapes, while also providing a deeper understanding of these issues.

3. Results and Discussion

Data collection was carried out by administering tests in the form of essay questions on solid geometry with flat surfaces and conducting interviews. After the researcher categorized the errors made by the students, the results were presented in a table for each question number.

Table 1 - Subject

No.	Subject	Question 1				Question 2				Question 3				Total
		Type of Error												
		1	2	3	4	1	2	3	4	1	2	3	4	
1.	AIA			✓	✓		✓	✓	✓		✓	✓	✓	8
2.	AI			✓	✓		✓	✓	✓			✓	✓	7
3.	ANW			✓	✓		✓	✓	✓	✓	✓	✓	✓	9
4.	AIB			✓	✓	✓	✓	✓	✓				✓	7
5.	MAA			✓	✓		✓	✓	✓	✓	✓	✓	✓	9
6.	KNA				✓				✓				✓	3
7.	MJ			✓	✓		✓	✓	✓	✓	✓	✓	✓	9
8.	NAT			✓	✓	✓	✓	✓	✓				✓	7
9.	SA			✓	✓		✓	✓	✓		✓	✓	✓	8
10.	SD			✓	✓		✓	✓	✓		✓	✓	✓	8
11.	MMI			✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	10
12.	RA			✓	✓		✓	✓	✓			✓	✓	7
13.	MFH			✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	10
14.	SNA			✓				✓					✓	3
15.	SR			✓				✓					✓	3
16.	MRR			✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	10

Description: 1: Factual Errors; 2: Conceptual Errors; 3: Principle Errors; 4: Operational Errors

Based on Table 1 above, three students were selected as subjects and interviewed. The selection of subjects was based on their willingness to be interviewed by the researcher and the types of errors they made, categorized as high, medium, and low in answering the questions. The students who made errors in solving solid geometry with flat surfaces were MMI in the high category, AIB in the medium category, and KNA in the low category, chosen as the research subjects. The following are the initials of the subjects based on the students' error categories: Student Error Data in Answering Solid Geometry with Flat Surfaces Problems.

Table 2 - Selected Subject

No.	Subjek	Question 1				Question 2				Question 3				Total
		Type of Error												
		1	2	3	4	1	2	3	4	1	2	3	4	
1.	MMI			✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	10
2.	AIB			✓	✓	✓	✓	✓	✓				✓	7

3.	KNA	✓	✓	✓	3

Based on the research results, the discussion is outlined as follows: The first finding from this study reveals the types of errors made by students with different levels of mathematical abilities in solving geometry problems related to solid figures with flat surfaces. Students with a high level of errors tend to make factual errors, conceptual errors, and operational errors. Students with a medium level of errors also experience similar types of errors but at a lower level, while students with a low level of errors mostly make operational errors without factual or conceptual errors.

These findings suggest that students with weaker mathematical understanding face difficulties in interpreting and applying basic information, formulas, and calculation processes, reflecting deeper conceptual and procedural errors. This result is consistent with previous research by Khosroshahi and Rosli (2019), which found that factual and conceptual errors are often interconnected, especially when students are under pressure or have a weak foundational understanding. This study also supports the finding that operational errors are often caused by hasty calculations or a lack of attention to detail, as identified in the research by Mubarak et al. (2020).

The differences in the types of errors also correlate with cognitive load theory (Sweller et al., 2019), which states that students with higher cognitive loads tend to make factual and conceptual errors due to excessive mental effort. Conversely, students with lower cognitive loads but who complete tasks quickly are prone to operational errors when they overlook verifying their calculations.

Factors that may influence these results include students' prior knowledge, exam anxiety levels, and the quality of instruction received. It is possible that the difficulty level of the geometry problems also contributed to the conceptual errors experienced by students with high errors, while students with low errors tend to feel confident and rush through their calculations, leading to operational errors. These patterns indicate a complex interaction between cognitive, emotional, and instructional factors in solving mathematical problems.

The second finding of this study identifies four main types of errors made by students in solving mathematical problems: factual errors, conceptual errors, principle errors, and operational errors. In general, these findings are consistent with recent literature showing that mathematical errors can occur at various levels of cognitive skill.

Factual Errors, caused by students' carelessness in reading the question or their inability to find the necessary information, indicate that understanding the problem is a major obstacle in problem-solving. A study by Carpenter et al. (2015) emphasized the importance of comprehension skills in context-based math problems, where students often struggle to extract relevant information from the question.

Conceptual Errors, caused by a lack of understanding of the tested concept or difficulty in selecting the appropriate formula, support the literature indicating that misconceptions are one of the main causes of failure in solving math problems. These issues stem from a lack of conceptual understanding and difficulty in visualizing the data in connection with the relevant geometric concepts (Kania et.al, 2024). Research by Star et al. (2020) shows that students often have partial or incorrect understandings of mathematical concepts, leading to errors in choosing problem-solving strategies.

Principle Errors, caused by students' inability to follow systematic problem-solving steps, reinforce the findings of Alibali et al. (2019), who found that procedural errors often occur when students do not understand the steps that need to be taken in sequence. This also highlights a gap in critical thinking and problem-solving skills, which are a focus of current educational curricula.

Operational Errors, such as calculation mistakes or over-reliance on calculators, support recent studies by Booth et al. (2021), which found that basic arithmetic skills and operational understanding remain major challenges for many students, especially in the context of solving more complex problems. The cause of students' low understanding is that they need to be more precise in solving problems and experience many mistakes, especially in choosing calculation operations and applying concepts (Valentine et.al., 2024).

These findings have several significant theoretical and practical implications. Theoretically, they support the error model in mathematics learning, where students tend to make errors due to both limitations in conceptual understanding and procedural skills. These findings also strengthen the literature indicating that students' ability to solve math problems is closely linked to their ability to understand the question, select the appropriate strategy, and apply mathematical principles effectively (Lamon, 2017).

From a practical perspective, these results suggest that educational interventions should focus on developing comprehension skills, both in reading questions and understanding underlying concepts. More interactive and contextual learning approaches, as suggested by Reeve et al. (2020), can help students improve their skills in tackling complex math problems. The use of educational technology, such as digital learning platforms, has also been shown to be effective in helping students understand difficult mathematical concepts through visual representations and real-time feedback (Schoenfeld, 2020).

In addition, this study suggests that math instruction should include more exercises focused on correct procedural understanding, as proposed by Fyfe et al. (2018). Teachers should also pay more attention to common operational errors, such as calculation mistakes, and encourage students to be more meticulous in checking their work.

This research contributes significantly to the theoretical understanding of students' error patterns in geometry, particularly in solving problems related to solid figures with flat surfaces. These findings highlight the multi-dimensional nature of students' errors and reinforce existing theories about cognitive load (Sweller et al., 2019). These findings also suggest that interventions aimed at reducing factual and conceptual errors need to address not only cognitive skills but also instructional delivery and problem presentation to reduce students' cognitive load.

One potential new theoretical contribution is the differentiation of error types based on students' ability levels. While previous research has focused more on conceptual errors (Khosroshahi & Rosli, 2019) or operational errors (Mubarak et al., 2020), this study connects both by showing that conceptual errors often precede operational errors in students with high levels of error. This progression of errors may contribute to the development of more detailed educational theories on how students process math problems at various stages of understanding.

From a practical perspective, these findings suggest that educators should tailor teaching methods to different levels of student understanding. For students prone to factual and conceptual errors, teachers can focus on improving basic understanding and reducing cognitive load by simplifying information and using more structured problem-solving techniques. Meanwhile, for students who mainly make operational errors, the focus should be on encouraging accuracy and self-checking mechanisms during problem-solving.

Moreover, these insights can inform curriculum design and teaching strategies that emphasize structured learning. Given that factual and conceptual errors are more common among lower-performing students, remedial programs targeting these areas could be developed. Meanwhile, for stronger students, providing more challenging problems and encouraging accuracy through timed practice can help minimize operational errors.

Despite generally aligning with previous studies, some differences were observed. For instance, the dominance of operational errors in students with low error levels has not been emphasized as much in previous literature. This difference may be due to variations in the student population or the type of geometry problems used in this study. Future research could explore these differences further by examining whether operational errors remain consistent across different types of math problems or if this only occurs in the context of solving specific geometry problems. Thus, this study not only confirms previous research but also extends it by providing a more detailed analysis of students' errors in geometry. The findings reveal the importance of considering the interaction between cognitive skills and problem-solving approaches and suggest that future interventions may be more effective by employing more structured and differentiated teaching strategies.

Despite these findings, there are also research limitations that need to be addressed. First, the sample used in this study was limited to one group of students at one school, which may limit the generalizability of these findings to a wider population of students. This limitation has been acknowledged in studies by Schneider and Preckel (2017), who emphasized the importance of testing findings with more diverse populations to increase external validity. Second, this study used only written tests as a data collection method. While these tests are useful for identifying the types of errors students make, this approach does not provide deeper insight into students' thinking processes during problem-solving. Research by De Koning et al. (2019) suggests that qualitative methods, such as interviews or direct observations, can provide richer information about the problem-solving strategies students use.

Therefore, based on these limitations, future research is recommended to involve larger and more diverse samples and to use more comprehensive data collection methods, including qualitative approaches. Further research is also needed to explore how technology-based educational interventions can help students overcome the various types of errors identified in this study. Additionally, future studies should focus on developing learning models that can improve students' understanding of complex mathematical concepts. Longitudinal studies are also needed to understand how students' error patterns develop over time and how long-term interventions can affect their learning outcomes in mathematics.

4. Conclusions

This study identifies the types of errors made by students in solving problems related to solid geometry with flat surfaces and the factors that contribute to them. Students with lower mathematical abilities tend to make factual, conceptual, and operational errors, while students with higher abilities primarily make operational errors. These findings suggest that weak conceptual understanding and high cognitive load are the main causes of errors. The study achieves its main objective by identifying distinct error patterns among students and clarifying the relationship between conceptual and operational errors. Theoretically, this research reinforces Van Hiele's theory of geometric development and supports cognitive load theory, especially in students facing complex problems. Practically, these results highlight the need for teachers to focus on both conceptual and procedural understanding while reducing students' cognitive load through more interactive and manipulative learning methods. The study's limitations include a small sample size and unstructured interviews, which may introduce bias. Future research should involve a larger sample and employ more comprehensive methodological approaches, such as direct observation. Overall, this research makes an important contribution to improving the understanding of student errors in geometry. The findings can be used to develop more effective teaching strategies and serve as a foundation for further studies and improvements in mathematics education practices in schools.

References

Alibali, M. W., Nathan, M. J., & Fujimori, Y. (2019). Learning mathematics in the era of cognitive science. *Cognitive Science, 43*(6), e12748. https://doi.org/10.1111/cogs.12748

Asipi, L. S., Rosalina, U., & Nopiyadi, D. (2022). The analysis of reading habits using Miles and Huberman interactive model to empower students' literacy at IPB Cirebon. *International Journal of Education and Humanities*, *2*(3), 117-125.

Booth, J. L., Barbieri, C., Eyer, F., & Paré-Blagoev, J. (2021). Comparing worked example and problem-solving instruction for improving algebraic equation solving. *Journal of Educational Psychology, 113*(4), 754-771. https://doi.org/10.1037/edu0000655

Carpenter, T. P., Franke, M. L., & Levi, L. (2015). Thinking mathematically: Integrating arithmetic and algebra in elementary school. *Journal of Mathematics Teacher Education, 18*(1), 1-9. https://doi.org/10.1007/s10857-014-9280-1.

De Koning, B. B., & van der Schoot, M. (2019). Learning by doing versus learning by viewing: Comparing two instructional approaches for solving geometry problems. *Learning and Instruction, 62*, 1-10. https://doi.org/10.1016/j.learninstruc.2019.02.004

Fyfe, E. R., DeCaro, M. S., & Rittle-Johnson, B. (2018). The relation between conceptual understanding and procedural skill: Does it matter how you measure it? *British Journal of Educational Psychology, 88*(1), 13-27. https://doi.org/10.1111/bjep.12181

Haser, Ç., & Star, J. R. (2020). Preservice mathematics teachers' knowledge and implementation of multiple strategies in the classroom. *Mathematical Thinking and Learning, 22*(2), 105-129.

Kania, N., Kusumah, Y. S., Dahlan, J. A., Nurlaelah, E., & Kyaruzi, F. (2024). Decoding Student Struggles in Geometry: Newman Error Analysis of Higher-Order Thinking Skills. *International Journal of Geometry Research and Inventions in Education (Gradient)*, *1*(01), 31–47. https://doi.org/10.56855/gradient.v1i01.1146

Khosroshahi, L. G., & Rosli, R. (2019). The nature of students' conceptual and procedural errors in solving mathematical word problems. *International Journal of Mathematical Education in Science and Technology, 50*(7), 1070-1083. https://doi.org/10.1080/0020739X.2018.1533674

Lamon, S. J. (2017). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers.* Routledge.

Litke, E. G. (2019). The nature and quality of algebra instruction: Using a content-focused observation tool as a lens for understanding and improving instructional practice. *Cognition and Instruction, 37*(3), 273-301.

Mason, M. (2018). Mathematical thinking in the classroom: A critical analysis of the Van Hiele theory. *Journal of Mathematical Behavior, 52*, 157-165.

Mubarak, H., Nordin, N., & Aman, R. (2020). Exploring the errors in solving algebraic problems among high school students. *Journal of Mathematics Education, 11*(3), 457-472. https://doi.org/10.1080/17940031.2019.1584543

OECD. (2018). *PISA 2018 results: Combined executive summaries.* OECD Publishing.

Reeve, J., Jang, H., & Carrell, D. (2020). Enhancing students' engagement by increasing teachers' autonomy support. *Journal of Educational Psychology, 112*(3), 499-518. https://doi.org/10.1037/edu0000412

Schneider, M., & Preckel, F. (2017). Variables associated with achievement in higher education: A systematic review of meta-analyses. *Psychological Bulletin, 143*(6), 565-600. https://doi.org/10.1037/bul0000098

Schoenfeld, A. H. (2020). Learning to think mathematically: Problem-solving, metacognition, and sense-making in mathematics. *Journal for Research in Mathematics Education, 51*(4), 300-324. https://doi.org/10.5951/jresematheduc-2020-015

Sweller, J., Ayres, P., & Kalyuga, S. (2019). *Cognitive load theory.* Springer.

Valentine, V. E., Alawiyah, T., Dewi, I. L. K., & Nopriana, T. (2024). Identifying the Mathematical Concept Understanding Ability of Junior High School Students in Learning Linear Function Material. *International Journal of Contemporary Studies in Education (IJ-CSE)*, *3*(2), 115-126. https://doi.org/10.56855/ijcse.v3i2.994

Van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education.* Academic Press.

Wijaya, A., Doorman, M., & Keijer, T. (2019). Difficulties in solving context-based PISA mathematics tasks: An analysis of students' errors. *Mathematics Education Research Journal, 31*(4), 351-371.