



Empowering Students with Discovery Learning in Circle Geometry for Better Problem-Solving

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Abstract

The objective of discovery-based teaching aids that use circular material is to assist young individuals in improving their mathematics skills. The ADDIE technique, which encompasses analysis, design, development, implementation, and evaluation, is considered during development. The analysis aims to ascertain the specific knowledge and skills students desire to acquire and the most effective instruction methods. Learning about the circle and the components that make it up and then applying that knowledge to the problem-solving process is one of the goals. Learning approaches focused on exploration are established during the project's design phase. In order to facilitate learning about the circular notion, visual aids and environments are currently being developed—the development process results in the production of learning modules, student workbooks, and interactive instructional materials. The designs determine the final result. The implementation process entails analysing learning, interactions between students, and the materials used in the classroom. Every effective method of evaluation incorporates the results of student learning and input from both the instructor and the students. This strategy aims to discover problems with the quality of the course content. According to the findings of several studies, students who participate in discovery-based training demonstrate improvements in their understanding of circles and their ability to solve mathematical problems independently. Involving kids in the learning process makes it more exciting and enjoyable.

Keywords: Circle topic, Development of teaching materials, Mathematical problem-solving.

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1. Introduction

An early introduction to geometry significantly enhances a child's ability to solve mathematical issues. The study by Rosadi et al. (2018) shed light on the significance of visualization, analysis, and informal deduction in geometry problem-solving. Additionally, they emphasize the importance of students acquiring the ability to engage in mathematical thinking and examine geometric figures to solve problems. Fauzi et al. (2018) intend to enhance the learning process of mathematical creative thinking in geometry by employing a metacognitive approach. This strategy can significantly improve students' problem-solving skills, specifically in geometry. Furthermore, Safrina et al. (2022) emphasize the potential of spatial training to enhance students' proficiency in solving geometric problems. This comment underscores the significance of spatial awareness and logical thinking when dealing with geometric challenges (Schall et al., 2009). Collectively, these studies emphasize the significance of geometry in the mathematics curriculum for fostering students' ability to think critically and solve problems.

To assist students in becoming more proficient in solving mathematical problems, we are utilizing the Discovery learning technique to design lesson plans for circle-based topics. It is consistent with the notion that solving mathematical issues needs higher-level cognitive capacities (Situmorang et al., 2022) that students draw on their prior knowledge to come up with novel solutions to a variety of problems. This method takes into account the fact that students rely on their previous knowledge. According to Nurlaelah et al. (2021), the capacity to solve mathematical problems effectively is regarded as an essential talent in mathematics education. As a result, mathematical education programs must incorporate tactics and approaches explicitly tailored for elementary school students (Kania et al., 2022). The purpose of this resource is to offer students an understanding of the significance of establishing individualized strategies and methods for the purpose of enhancing their ability to solve mathematical problems. encourages pupils to develop the ability to solve problems as part of their maths education effectively (Kamid et al., 2023)

Sumarni et al. (2022) state that instructional materials can be produced and implemented more effectively if they adhere to the ADDIE paradigm, which guarantees a methodical approach from analysis to evaluation. The ADDIE model is a framework for the systematic design of learning that consists of five phases: analysis, design, development, implementation, and evaluation. The (Lawe et al., 2021). This model is commonly utilised in educational settings for the purpose of developing effective instructional materials. It does this by assuring an organized approach from the analysis's beginning to the evaluation's conclusion (Bauyot, 2024). Educators can create well-analyzed, designed, implemented, and evaluated materials following the ADDIE model (Putri & Amini, 2023). This ultimately results in an improvement in the teaching and learning experience when it is implemented. According to research by Pamugari et al. (2020), the ADDIE model offers a regimented approach to developing instructional materials. This approach guarantees comprehensive analysis, design, development, implementation, and assessment to enhance education outcomes. According to research by Puspitasari et al. (2020), the ADDIE model incorporates stages such as analysis, development, implementation, and assessment, which are highly significant activities in developing effective instructional materials. According to research by Sitepu et al. (2020), the ADDIE model is widely acknowledged as an acceptable framework for developing technology-assisted learning materials that contribute to improving blended learning experiences.

A further demonstration of the significance of the practical application of information and hands-on learning in mathematics education is provided by utilizing the Polya problem-solving approach, which has been acknowledged as an efficient strategy for enhancing the problem-solving abilities of students (Hadi & Radiyatul, 2014). Enhancing students' ability to solve mathematical issues and think critically can be achieved by using a variety of learning tactics, such as discovery learning, problem-based learning, technology integration, systematic learning design models, and strategies that have been proven to be effective in solving problems. On the topic at hand are circles.

2. Methods

This research is a Research and Development (R&D) project that focuses on creating instructional resources using discovery steps for circle topics. This study uses the ADDIE development model, a versatile research framework suitable for development research. The ADDIE approach comprises five stages: analysis, design, development, implementation, and evaluation. The analysis stage evaluates the need to create new items (such as models, methods, media, and training materials) and assesses the practicality and prerequisites for product development. The design stage is a methodical procedure that begins with developing the product concept and content. The development stage includes the actualization of products that have been previously conceived. The Implementation Phase includes the practical application of the product within the ADDIE development research paradigm to gather feedback regarding existing or developed products. The evaluation stage offers feedback to product users, allowing adjustments based on evaluation results or unmet product requirements. However, this study mainly focuses on the development stage, as other stages were excluded due to time constraints. Figure 1 displays the sequential phases of ADDIE.

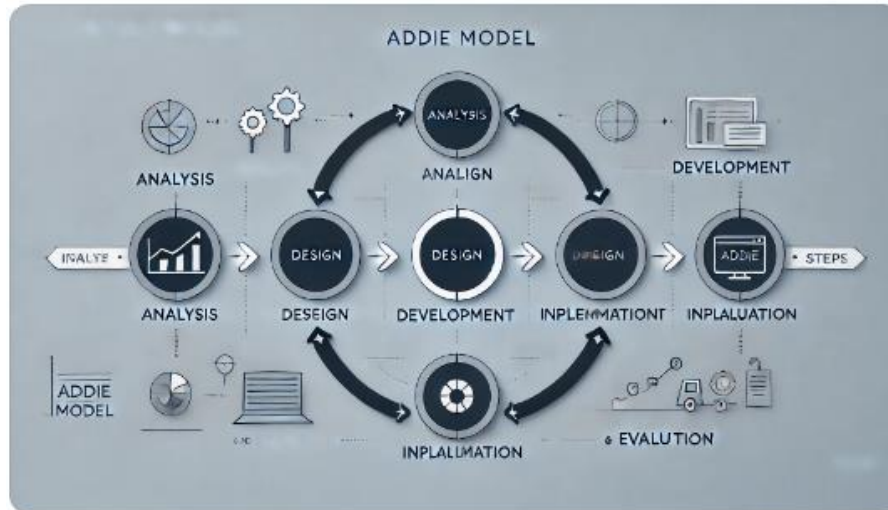


Figure 1 The stages of the ADDIE model.

3. Results and Discussion

This study generates reliable instructional resources on circles. These resources are specifically crafted to enhance proficiency in mathematical problem-solving. The development of this teaching material follows the ADDIE approach, which is exclusively focused on the development stage. The results of this investigation are elucidated in the subsequent stages of development.

3.1.1. Analysis stage

The initial phase of the ADDIE development paradigm is analysis. During this stage, analysis is conducted to determine the necessity of developing a new product and to analyze the practicality and prerequisites for product development. According to the study conducted at SMP Islam Terpadu at-Tadzkir Majalengka, the observations indicate a strong need to develop teaching materials to enhance mathematical problem-solving skills, specifically circular material. This need is evidenced by the subpar exam scores of three capable kids across varying ability levels.

The exam results indicate that only students with a high proficiency level can answer the questions with considerable success. This suggests that many students have not yet grasped the content completely. Moreover, interviews were conducted with students, revealing that their limited abilities were primarily attributed to a deficiency in comprehending concepts when solving mathematical issues. The limited capacity to solve mathematical issues results from instructional materials failing to adequately foster and enhance problem-solving skills.

According to teacher interviews, using exclusive textbooks as the sole teaching resource resulted in student weariness and apathy, particularly in mathematics. Hence, creating more engaging instructional resources is imperative to enhance students' proficiency in solving mathematical problems. Researchers will create educational resources centered around circular themes, incorporating progressive learning stages.

3.1.2. Design Stage

The ADDIE research and development model activities adhere to a systematic approach that commences with designing the product's concept and content. This stage is the first phase of the process, where instructional materials are created using the discovery approach in circular material. It involves identifying and obtaining materials, selecting the appropriate application, and detailing the production process.

The phases involved in the ADDIE model, a methodological approach to learning design, are represented visually in Figure 2. To gain knowledge about the learning context, the first step is to conduct an analysis, including determining needs, goals, objectives, and goals. The next step is called design, and it is in this step that you will select appropriate resources and establish your teaching strategy (Kania & Juandi, 2023). After that, the next stage is the development stage, where all the information is used as a basis for development. Immediately after that is the implementation period, namely when the learning developed is taught to students. For the final evaluation step, feedback is provided regarding how effectively and efficiently the training process has been implemented. A comprehensive and effective learning design method is guaranteed by the logical development of each phase directly into the next.

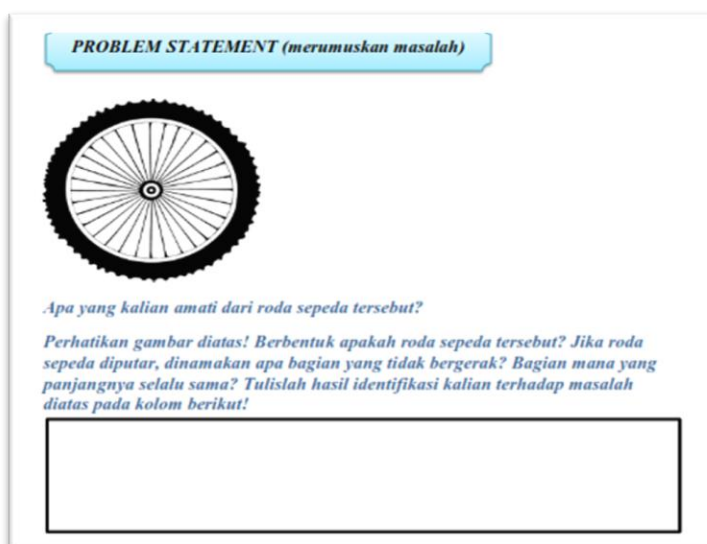
The source material utilized consists of textbooks aligned with the prescribed curriculum. The design process involves creating educational materials that follow a series of discovery processes. These steps include summarising and organizing the content to align with the discovery process. Figure 2 displays the outcomes obtained from creating educational resources incorporating a series of discovery-based activities within the circular material that has been produced. The following are circle teaching materials that were developed following discovery steps to increase mathematical problem-solving abilities:



In everyday life, we encounter many objects whose surface is circular. Circles, such as technology, sports, architecture, coins, etc, are helpful in life. On the field, we also often encounter circular technology, such as car wheels, wheels motorbikes, cassettes, car steering wheels, and so on

Figure 2 Remembering steps

This page does a fantastic job of introducing children to the concept of circular elements, which is the main objective of the page, which is to teach children how to distinguish the components that make up a circle. The "Ayo Mengingat!!!" section asks students to reflect on the information they already know. When the lesson is connected to everyday things like technology, sports, architecture, coinage, and various technological components like automobile wheels and CDs, it becomes more approachable and valuable to the students. A picture of a bicycle is an excellent visual aid since it demonstrates how circles can be utilized in the most commonly encountered scenario. Even though the language is well-structured and simple to read, the list of examples might be improved by adding additional white space or bullet points to make them more readable (Ling et al., 2021; Uegatani et al., 2021). The page is well-organized and explains the concept of circular elements using visual and contextual features (Mudaly, 2021). Additionally, the page is neatly ordered.



What observations can you make regarding bicycle wheels? Observe the image positioned above! What is the geometric shape of bicycle wheels? If a bicycle wheel is turned, what is the stationary part called? Which specific section or component are you referring to? Is the length consistently uniform? Record the outcomes of your problem identification in the adjacent column.

Figure 3 Problem statement

Using a bicycle wheel as a metaphor, this page instills in children an understanding of issue statements while encouraging critical thinking and observation. Developing and recognizing difficulties is the activity objective the students will participate in. This objective is abundantly obvious by the prompt "MASALAH 1.1: PROBLEM STATEMENT (merumuskan masalah)," which includes the same name. Students can strengthen their analytical skills as they work through the questions designed to assist them in analyzing the wheel's form, motion, and fixed components (Kania et al., 2023). Students can better learn and grasp abstract concepts with the assistance of the visual of the bicycle wheel, which in turn makes them more invested in the action that is now being performed. The well-structured design that fosters methodical thought and careful data collection includes a designated place for students to record their discoveries and observations. This section is part of the design (Ballard & Dymond, 2023). This strategy enhances one's grasp of circular elements and encourages the development of problem-solving skills within a realistic and applicable context.

DATA COLLECTION (pengumpulan data)

Setelah mengidentifikasi masalah diatas, tuliskan pada kolom dibawah ini informasi apa saja yang kalian temukan!

If you have discovered the problem mentioned earlier, please enter it in the column below. Include any relevant information.

Figure 4 Data collection steps

Data collection is much easier when students have a consistent place to write their observations. With the heading "DATA COLLECTION," the purpose of this task is laid out. The sentence "After identifying the problem above, write down in the column below what information you find!" encourages students to submit their results after identifying the problem. Data gathering and organisation are the focal points of this approach. The ample writing space promotes the use of more detailed remarks. Scientific inquiry and problem-solving rely on analytical reasoning and meticulous record-keeping, all of which are encouraged on this website.

DATA PROCESSING (pengolahan data)



Perhatikan kembali gambar roda sepeda! Berdasarkan data yang telah kalian kumpulkan, dapatkan kalian temukan unsur-unsur lingkaran dari gambar diatas?

Jawab :

- Titik O disebut dengan titik . . .
- Garis OA, OB dan OC disebut dengan . . .
- Garis AC disebut dengan . . .
- Garis AB dan garis CD disebut dengan . . .
- Garis lengkung AB disebut dengan . . .
- Daerah arsiran yang dibatasi oleh titik BOC disebut dengan . . .
- Daerah arsiran yang dibatasi oleh titik AB disebut dengan . . .
- Garis OE disebut dengan garis . . .

Look again at the picture of the bicycle wheel! Be aware of the data you have put it together, can you find the elements of the circle from the picture above?

Answer:

- Point O is called point . . .
- Lines OA, OB and OC are called. . .
- The AC line is called . . .
- Line AB and line CD are called. . .
- The curved line AB is called . . .
- The shaded area bounded by the BOC point is called . . .
- The shaded area bounded by point AB is called . . .
- Line OE is called line . . .

Figure 5 Data processing steps

This well-designed website, which focusses on circular element detection, guides data processing. The "DATA PROCESSING (pengolahan data)" section clearly states the activity's goal. The segmented and labeled bicycle wheel schematic in the presentation will help students understand abstract topics. The directions require pupils to return to the illustration and apply their knowledge to identify which parts of the circle belong to which parts. Students can demonstrate their understanding of center, radii, diameter, chords, arcs, and sectors by methodically solving the puzzles. This systematic technique simplifies geometry comprehension and retention (Erdogan, 2019; Wahyudi et al., 2020). Bullet points to organise questions and a neat manner make the procedure seem less scary. Finally, this page's analytical tasks and visual learning tools help readers grasp circular geometry.

VERIVICATION (pembuktian)

Setelah kamu mengetahui unsur-unsur lingkaran. Sekarang perhatikan garis AC dan AO, apakah terdapat hubungan antara garis tersebut? Tuliskan pada kolom berikut!

Once you are familiar with the components of a circle. Now let's examine the AC line and AO. Is there a correlation between these lines? Enter the information in the column that comes after this one.

Figure 6 Verification steps

Students can follow the well-organised page during the verification step, which teaches circle line segment relationships. The exercise's "VERIFICATION (pembuktian)" component specifies its goal. After studying the different sections of a circle, students are instructed to research the link between AC and AO. By connecting theoretical principles to real examples, this strategy promotes analytical thinking and improves students' knowledge. Since there is a lot of vacant space for students to record their findings, they should provide detailed explanations and observations. This website integrates theoretical material with practical assessment to help students improve their analytical skills (Kania et al., 2023; Masuku et al., 2020). Students may easily follow and engage because the structure is well-organised and the directions are simple.

GENERALIZATION (kesimpulan)

Dari permasalahan 1.1, jelaskan unsur-unsur yang kamu temukan pada lingkaran dengan bahasamu sendiri. Tuliskan pada kolom dibawah ini!

From problem 1.1, explain the elements you found in circle in your own language. Write it in the column below!

Figure 7 Generalization steps

Using this page, students can apply what they have taught about circle components to a more general setting. The "GENERALISATION (kesimpulan)" component teaches students how to synthesize and apply information. To demonstrate that they have understood the material, students are encouraged to define the components of circles using their own words. This exercise helps them remember and comprehend the material better. There is sufficient writing space for students to build on their arguments.

Within the practical issue that can be found at the bottom of the page, students are strongly encouraged to apply their knowledge to a particular objective (Ratnawulan & Kania, 2020; Sönmez & Alptekin, 2020). Creating a circle that satisfies the requirements and determining the circle's diameter is necessary to complete this task. The pupils can remember more information by putting what they have been taught into practice through this problem-solving task (Angraini et al., 2023; Rahman et al., 2021; Susilo et al., 2023) Pupils can participate in the exercise because both the design and the directions are straightforward. This article does a good job of summarising and applying the fundamentals of circular geometry, which makes it considerably simpler to comprehend and remember.

3.1.3. Development Stage

The subsequent phase is the development stage, encompassing the validation process. This step aims to generate the ultimate version of the teaching materials generated thus far in the design stage. Specialists have verified the teaching materials containing circular material discovery phases, and the outcomes are presented in Table 1.

Tabel 1. Validation results from experts

Validator	Score	Interval	Categories
1	47	$35 < x < 52$	Valid
2	42		Valid
3	40		Valid
4	45		Valid

According to the validity study conducted by specialists, it has been demonstrated that the validity assessment for teaching materials with a discovery stage in circular materials falls into the "valid" category. The validation criteria ascertain the high quality and dependability of the teaching materials specifically designed for teaching mathematics to 8th-grade junior high school pupils in the classroom.

4. Conclusions

The teaching materials on circle material satisfy the valid criteria as per the stages of the validity test. These materials are suitable for classroom use, particularly for teaching mathematics to junior high school students in the eighth grade. This teaching material can be an alternative lesson plan for teachers and students. Future researchers are advised to perform additional research to assess students' mathematical ability enhancement by utilizing this instructional material. In addition, it is strongly advised to conduct additional studies to incorporate technology into the delivery process effectively.

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Conflict of Interest

The authors declare no conflicts of interest.

References

- Angraini, L. M., Larsari, V. N., Muhammad, I., & Kania, N. (2023). *Generalizations and analogical reasoning of junior high school viewed from bruner's learning theory*. 12(2), 291–306.
- Ballard, S. L., & Dymond, S. K. (2023). Beliefs About Secondary-Age Students with Extensive Support Needs Participating in Their Health Care at School. *Research and Practice for Persons with Severe Disabilities*, 48(2), 92–107. <https://doi.org/10.1177/15407969231173932>
- Bauyot, M. M. (2024). Developing an ADDU Online Pedagogical Framework. *Journal of Electrical Systems*, 20(45), 1448–1457. <https://doi.org/10.52783/jes.2188>
- Erdogan, F. (2019). Effect of cooperative learning supported by reflective thinking activities on students' critical thinking skills . *Eurasian Journal of Educational Research*, 2019(80), 89–112. <https://doi.org/10.14689/ejer.2019.80.5>
- Hadi, S., & Radiyatul, R. (2014). Metode Pemecahan Masalah Menurut Polya untuk Mengembangkan Kemampuan Siswa dalam Pemecahan Masalah Matematis di Sekolah Menengah Pertama. *EDU-MAT: Jurnal Pendidikan Matematika*, 2(1). <https://doi.org/10.20527/edumat.v2i1.603>
- Kamid, Ramalisa, Y., Fajriah, N., Kurniawan, D. A., Perdana, R., Rivani, P. A., Widodo, R. I., & Ningsih, I. W. (2023). Collaborative Character, Cognitive Psychology, and Process Skills: The Impact of Character-Based Mathematics Learning in High Grades of Elementary Schools. *Journal of Education Research and Evaluation*, 7(4), 596–607. <https://doi.org/10.23887/jere.v7i4.55095>
- Kania, N., Fitriani, C., & Bonyah, E. (2023). Analysis of Students' Critical Thinking Skills Based on Prior Knowledge Mathematics. *International Journal of Contemporary Studies in Education (IJ-CSE)*, 2(1), 49–58. <https://doi.org/10.56855/ijcse.v2i1.248>
- Kania, N., & Juandi, D. (2023). Does self-concept affect mathematics learning achievement ? 17(3), 455–461. <https://doi.org/10.11591/edulearn.v17i3.20554>
- Kania, N., Sudianto, S., & Hanipah, H. (2022). Analysis of Student's Geometry Thinking Ability Based on Van Hiele's Theory. *Journal of Mathematics and Mathematics Education*, 12(1), 21–33. <https://doi.org/10.20961/jmme.v12i1.62273>
- Lawe, Y. U., Noge, Desidaria, M., & Rato, Kristoforus Portasius Dosi Novaliendry, D. (2021). Creation Of Multilingual Teaching Materials Focused On Content And Background Of Ngada Culture For Primary 1st Grade. *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, 12(2), 3110–3118. <https://doi.org/10.17762/turcomat.v12i2.2355>
- Ling, G., Elliot, N., Burstein, J. C., McCaffrey, D. F., MacArthur, C. A., & Holtzman, S. (2021). Writing motivation: A validation study of self-judgment and performance. *Assessing Writing*, 48(July 2020), 100509. <https://doi.org/10.1016/j.asw.2020.100509>
- Masuku, M. M., Jili, N. N., & Sabela, P. T. (2020). Assessment as A Pedagogy and Measuring Tool in Promoting Deep Learning In Institutions of Higher Learning. *International Journal of Higher Education*, 10(2), 274. <https://doi.org/10.5430/ijhe.v10n2p274>
- Mudaly, V. (2021). Constructing mental diagrams during problem-solving in mathematics. *Pythagoras*, 42(1), 1–8. <https://doi.org/10.4102/PYTHAGORAS.V42i1.633>
- Nurlaelah, A., Ilyas, M., & Nurdin. (2021). Pengaruh Adversity Quotient Terhadap Kemampuan Pemecahan Masalah Matematis Siswa SD. *Proximal: Jurnal Penelitian Matematika Dan Pendidikan Matematika*, 4(2), 89–97. <https://doi.org/10.30605/proximal.v4i2.1367>
- Putri, V. M., & Amini, R. (2023). Integrated Thematic E-LKPD with RADEC- Based Neapod in Grade V Elementary School. *International Journal of Elementary Education*, 7(2), 204–211. <https://doi.org/10.23887/ijee.v7i2.61224>
- Rahman, N. A., Rosli, R., Rambely, A. S., & Halim, L. (2021). Mathematics teachers' practices of stem education: A systematic literature review. *European Journal of Educational Research*, 10(3), 1541–1559. <https://doi.org/10.12973/EU-JER.10.3.1541>

- Ratnawulan, N., & Kania, N. (2020). Implementation of Cooperative Learning Model Numbered Head Together (Nht) Type To Improve Learning Activities. *Jurnal THEOREMS (The Original Research of Mathematics)*, 4(2), 161–168.
- Schall, G., Mendez, E., Kruijff, E., Veas, E., & ... (2009). Handheld augmented reality for underground infrastructure visualization. *Personal and Ubiquitous ...*. <https://link.springer.com/content/pdf/10.1007/s00779-008-0204-5.pdf>
- Situmorang, D. F., Siahaan, T. M., & Tambunan, L. O. (2022). Pengaruh Model Pembelajaran Treffinger Terhadap Kemampuan Pemecahan Masalah Matematis. *Jurnal Pendidikan Sains Dan Komputer*, 2(02), 396–404. <https://doi.org/10.47709/jpsk.v2i02.1793>
- Sönmez, N., & Alptekin, S. (2020). Teaching a Student with Poor Performance in Mathematics to Recall of Multiplication Facts Using Simultaneous Prompting with Systematic Review and Corrective Feedback. *World Journal of Education*, 10(3), 33. <https://doi.org/10.5430/wje.v10n3p33>
- Sumarni, S., Adiastuty, N., Riyad, M., Fisabqi, N., & Pritiya, W. (2022). Development of Geometry Teaching Materials Assisted Geogebra Android to Improve Mathematical Connection Ability. *Proceedings of the 2nd Universitas Kuningan International Conference on System, Engineering, and Technology, UNISSET 2021, 2 December 2021, Kuningan, West Java, Indonesia*. <https://doi.org/10.4108/eai.2-12-2021.2320201>
- Susilo, F. J., Usodo, B., & Sari, D. (2023). The Profile of High-order Thinking Skills of Junior High School Students. *International Journal of Mathematics and Mathematics Education*, 1(1), 77–82. <https://doi.org/https://doi.org/10.56855/ijmme.v1i1.266>
- Uegatani, Y., Nakawa, N., & Kosaka, M. (2021). Changes to Tenth-Grade Japanese Students' Identities in Mathematics Learning During the COVID-19 Pandemic. *International Electronic Journal of Mathematics Education*, 16(2), em0638. <https://doi.org/10.29333/iejme/10905>
- Wahyudi, W., Waluya, S., Suyitno, W., & Isnarto, I. (2020). The impact of 3CM model within blended learning to students' creative thinking ability. *Journal of Technology and Science Education*, 10(1), 32. <https://doi.org/10.3926/jotse.588>
- Z, R., Nuraina, N., & Novia, N. (2022). PENDEKATAN CONTEXTUAL TEACHING AND LEARNING UNTUK MENINGKATKAN KEMAMPUAN PEMECAHAN MASALAH MATEMATIS DAN MINAT BELAJAR. *Jurnal Pendidikan Matematika Malikussaleh*, 2(2), 285. <https://doi.org/10.29103/jpmm.v2i2.9268>