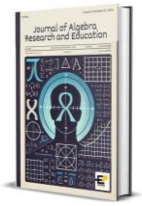
EDUPEDIA Publisher

INTERNATIONAL JOURNAL
APPLIED LEARNING AND RESEARCH IN ALGEBRA
Vol. 02 No. 2 (2025), 96-105
https://journals.eduped.org/index.php/Algebra
E-ISSN 3063-9824



Students' Errors in Solving Algebraic Dominant Numeration Problems Based on Kastolan Theory

Pitri Suryani[1], Nurrahmawati[2], Lusi Eka Afri[3], Arcat[4*]

[1,2,3,4]Mathematics Education, Universitas Pasir Pengaraian, Indonesia

*Corresponding author: Arcat86@gmail.com

Article Info

Received February 19, 2025
Revised April 22, 2025
Accepted June 17, 2025

Abstract

Minimum competency assessment is a basic competency that all students need to be able to develop their own abilities and play an active role in society in positive activities. Minimum competency assessment aims to measure students' cognitive learning outcomes which include reading literacy and numeracy competencies. This study aims to determine the types of errors made by students in solving algebraic domain numeracy problems based on the kastolan theory, namely: (1) conceptual errors, (2) procedural errors, (3) technical errors. The research that has been conducted is a qualitative descriptive study with a research instrument of three AKM type algebra essay questions on 26 class VIII 3 students at SMP N 1 Tambusai. The results obtained based on the results of student answer sheets include: (1) conceptual errors of 30.76%, procedural errors of 38.46%, and technical errors of 32.05%. It can be concluded that the mistakes that students often make are procedural errors. The results of this study provide an overview of the various errors made by students in solving algebraic domain AKM problems, so that they can determine follow-up actions so that students' numeracy skills become better.

Keywords: Algebraic Domain; Kastolan Theory; Numeration Problems Based.

To cite this article

Suryani, P., Nurrahmawati, Afri, L. E., & Arcat. (2025). Students' Errors in Solving Algebraic Dominant Numeration Problems Based on Kastolan Theory. *Journal of Algebra Research and Education,* 02(2), pp, 96-105, doi. https://doi.org/10.56855/algebra.v2i2.1491

1. Introduction

In 2021, the Ministry of Education and Culture of Indonesia officially replaced the National Examination with the National Assessment, marking a significant shift in the country's educational evaluation system (Ministry of Education and Culture, 2021). Unlike the National Examination, which emphasized mastery of subject-specific content, the National Assessment focuses on two fundamental competencies: reading literacy and numeracy. This transition aims to provide comprehensive insights into students' cognitive abilities to support the improvement of teaching and learning quality (Putra & Fitria, 2022; Yusuf et al., 2023).

The National Assessment comprises three main components: the Minimum Competency Assessment (Asesmen Kompetensi Minimum/AKM), the Character Survey, and the Learning Environment Survey (Kemendikbudristek, 2021). The AKM is designed to assess essential cognitive competencies that students must acquire to become effective members of society. These include reading literacy and numeracy, both of which are critical for lifelong learning and problem solving in real life contexts (Sari & Hidayat, 2022; Rahmawati et al., 2023).

Numeracy, in particular, is defined as the ability to apply mathematical knowledge and reasoning to solve problems in various contexts, including personal, occupational, and societal settings (Ningsih & Syahputra, 2023). The scope of numeracy includes four domains: number, measurement and geometry, data and uncertainty, and algebra (Wulandari & Hidayat, 2021). Algebra plays a central role in mathematics education, especially at the middle school level, where students are introduced to algebraic expressions and equations (Lestari et al., 2022). However, numerous studies have reported persistent challenges faced by students in mastering algebraic concepts due to the abstract nature of algebra and the prerequisite knowledge it demands (Hakim & Mariani, 2023).

Recent data from the National Assessment revealed a 6.68% decline in student performance in the algebra domain between 2022 and 2023, indicating a notable increase in student errors when solving algebra-based numeracy problems (Kemendikbudristek, 2023). Such errors are often attributed to low student motivation and engagement with mathematics, as well as limited conceptual understanding and procedural fluency (Fauzi & Irwandi, 2022).

Error analysis is a valuable approach for identifying, categorizing, and understanding the types of mistakes students make in mathematical problem solving (Ramdani & Fitriani, 2024). According to Kastolan (1992), mathematical errors can be classified into three main categories: conceptual errors (misunderstanding principles or definitions), procedural errors (incorrect application of algorithms or steps), and technical errors (computational or symbolic mistakes). By employing Kastolan’s framework, educators and researchers can gain deeper insights into the root causes of student difficulties in solving algebra problems.

Based on this background, this study aims to analyze the types of errors made by Grade VIII students at SMP Negeri 1 Tambusai when solving numeracy problems in the algebra domain. The analysis will use the Kastolan theory as a framework to describe and categorize students' mathematical errors, thereby contributing to efforts to enhance instructional strategies and student outcomes in mathematics education.

2. Methods

This study employed a qualitative research design with a case study approach. The primary objective was to explore and analyze the types of errors made by students in solving numeracy problems, specifically within the algebra domain, using Kastolan's theoretical framework. The qualitative approach was chosen because it enables a detailed and in-depth examination of students' thought processes, allowing the researcher to gain meaningful insights into the underlying causes of their mistakes.

The research was conducted at State Junior High School 1 Tambusai, focusing on students in the eighth grade. The selection of participants was purposive, based on the specific types of errors students made while completing a numeracy test designed for the study. The selection criteria aimed to ensure that a range of error types would be represented in the data, allowing for a comprehensive analysis. In total, six students were selected to participate in interviews. These students were categorised into three groups, each representing a different type of error: conceptual, procedural, and technical. Two students from each category were chosen, providing balanced representation and enabling the researcher to analyse patterns and causes of each error type in detail.

To collect data, the study utilized two main instruments: a set of numeracy test questions and a structured interview guide. The numeracy test consisted of six algebra-based questions designed to reveal common student errors. Before their use in the actual research, the test items underwent a trial phase with a group of students similar to the main research sample. This pilot testing served to evaluate the clarity, appropriateness, and difficulty level of each question. As part of the trial process, the test items were subjected to a validity assessment to ensure that they accurately measured what they were intended to measure. All six questions were deemed valid based on the results of this assessment.

In addition to validity, the reliability of the test was also evaluated to determine the consistency of the instrument. This was done using the Alpha formula, which is commonly used to calculate the reliability coefficient of a test. The resulting reliability coefficient (r11) was found to be 0.616. This value indicated that the test was sufficiently reliable for the study, providing a dependable tool for collecting data from the students.

The data collection process was conducted in two main stages. First, all participating students completed the numeracy test. The results of this test were then analysed to identify the types of errors made by the students, which served as the basis for selecting the interview participants. Following the test, structured interviews were conducted with the six selected students. The interviews were designed to probe deeper into the students' reasoning and thought processes, helping the researcher uncover the causes behind their mistakes and misunderstandings.

The analysis of the collected data followed a structured and systematic process consisting of four main stages: data collection, data reduction, data presentation, and conclusion drawing. During the data reduction stage, the researcher carefully examined the students' written responses and interview transcripts to identify recurring patterns and categorise the errors according to the types defined by Kastolan's theory. Conceptual errors were identified as misunderstandings or misinterpretations of mathematical principles or concepts. Procedural errors involved incorrect steps or misapplication of mathematical procedures, while technical errors included minor calculation mistakes or issues related to the notation and use of symbols.

Once the data were categorized, the results were presented in a detailed and organized manner to illustrate the specific nature of each type of error and highlight any common patterns observed among the students. The presentation included examples of student work, summaries of interview responses, and explanatory narratives that clarified the logic behind the students' answers. Conclusions were drawn based on the patterns and findings revealed through the analysis. These conclusions provided insights into the root causes of the errors and offered suggestions for educators on how to address these issues in the classroom. By identifying the specific challenges faced by students in the algebra domain, the study aims to contribute to the development of more effective teaching strategies and improve students' numeracy skills overall.
skills.

3. Results and Discussion

3.1 Results

This section presents the results of an analysis of students' written responses to numeracy test questions focused on the algebra domain. The responses were evaluated using Kastolan's error classification framework, which categorizes student mistakes into three types: conceptual errors, procedural errors, and technical errors. The study involved 26 eighth grade students from State Junior High School 1 Tambusai, and each student's work was analyzed to identify the nature and frequency of errors in solving algebraic problems. Each question in the test was designed to assess different cognitive levels: comprehension, application, and reasoning. The distribution of errors across these cognitive levels is critical in understanding how students process mathematical problems and where specific difficulties arise.

In Problem 1, which was designed to assess students' comprehension of algebraic concepts, the most prevalent error type was conceptual, accounting for 30.76% of the errors. This indicates that a significant number of students struggled with understanding the fundamental ideas behind algebraic expressions or operations. These conceptual errors included difficulties in recognizing appropriate formulas, misunderstanding algebraic terms, and an inability to correctly interpret problem instructions. Many students appeared to memorize formulas without fully grasping their meaning or how to apply them in various contexts, leading to incorrect solutions.

Problem 2, aligned with the application level of cognition, revealed a high rate of procedural errors, which comprised 38.46% of the mistakes observed. These errors occurred when students failed to carry out the correct sequence of steps needed to arrive at a solution. Some began with the correct approach but deviated midway due to confusion or uncertainty about the next step. Others skipped critical steps or used incorrect procedures entirely. This pattern suggests a lack of fluency in applying structured methods and may be attributed to limited practice, gaps in instruction, or weak problem solving strategies.

Problem 3, which targeted the reasoning level, also showed a dominance of procedural errors, making up 32.05% of student mistakes. While this problem demanded logical thinking and multi-step reasoning, many students struggled to maintain a coherent problem solving path. This again highlights weaknesses in procedural understanding, particularly in problems that require integration of multiple mathematical skills.

While technical errors which include basic arithmetic mistakes, misplacement of symbols, or careless miscalculations were present across all problems, they were not the most dominant in any single item. However, their presence remains significant, as they often led to otherwise avoidable incorrect answers. These technical errors typically stemmed from lapses in attention, rushing through calculations, or neglecting to review completed work.

A summary of these findings shows distinct characteristics for each type of error. Conceptual errors involve incorrect application of formulas, misunderstanding algebraic rules, and a superficial understanding of concepts. The causes of these errors include misconceptions, limited prior knowledge, and an over-reliance on memorizing rules without understanding their purpose. Procedural errors, on the other hand, are related to a failure in executing problem-solving steps in a logical and correct sequence. These often arise from incomplete procedural knowledge, poor time management, and a lack of structured practice. Lastly, technical errors are simple but impactful mistakes made during calculations, often resulting from carelessness, insufficient focus, or anxiety under timed conditions.

In conclusion, the data reveals that all three error types conceptual, procedural, and technical are present among students, each with distinct patterns and causes. Conceptual understanding appears to be the weakest link, especially when students are required to interpret and apply fundamental algebraic principles. Procedural knowledge is also lacking, especially in tasks requiring multi-step reasoning, while technical accuracy is undermined by inattention and anxiety. These findings provide a clear direction for educators to focus more on building strong conceptual foundations, reinforcing procedural fluency through structured problem-solving activities, and cultivating carefulness and confidence during mathematical tasks.

3.2 Discussion

The findings of this study highlight a significant trend among eighth grade students, who frequently experience difficulties in solving numeracy problems within the algebra domain due to conceptual, procedural, and technical errors. These three types of errors do not exist in isolation but often overlap, compounding the students' challenges in understanding and applying mathematical concepts effectively. The complexity and interdependence of these errors underscore the need for a holistic approach in addressing students' learning difficulties in mathematics. Conceptual errors emerged as the most common type of mistake, particularly in test items that required comprehension-level thinking. These errors typically stem from a fundamental misunderstanding of algebraic principles such as variable manipulation, the meaning of symbols, and the application of appropriate formulas. Many students appeared to depend heavily on memorization rather than truly understanding the underlying mathematical concepts. This reliance on rote learning led them to choose incorrect formulas, misinterpret problem contexts, and fail to establish logical relationships between algebraic expressions and the problems presented.

The prevalence of conceptual errors is concerning, as a strong conceptual foundation is essential for success in higher-level mathematics. Without it, students are less likely to develop flexible thinking or transfer their knowledge to unfamiliar problems. Research has shown that deficiencies in conceptual understanding significantly hinder students' ability to progress in mathematics and often result in long-term learning difficulties (Siregar & Fadilah, 2022; Wibowo et al., 2021). Moreover, when students lack sufficient exposure to algebra in earlier grades, their understanding of foundational concepts remains fragile, which further impedes their performance in more complex topics (Lestari et al., 2021; Rahmawati et al., 2023).

Procedural errors, on the other hand, were found most frequently in tasks that required the application of knowledge and logical reasoning. These errors occurred when students were unable to execute the correct sequence of steps or procedures needed to arrive at a solution. For instance, students might begin a problem correctly but deviate from the logical path midway through, suggesting confusion about the appropriate procedural flow. In other cases, students skipped essential steps or applied an incorrect algorithm. Such errors reflect a lack of fluency in mathematical procedures and an inability to connect one step logically to the next. The importance of procedural fluency, defined as the ability to apply procedures accurately, efficiently, and flexibly, is widely recognised in mathematics education (Hasanah & Fauzan, 2020). It complements conceptual understanding by enabling students to carry out mathematical tasks with confidence and accuracy. Students with weak procedural skills often require structured guidance to develop problem-solving strategies that are systematic and logical (Pertiwi & Nugroho, 2023). The absence of step-by-step reasoning instruction in classrooms could be a major contributing factor to the widespread procedural errors observed in this study.

While technical errors were less dominant than the other two categories, they were nonetheless evident across all test items. These errors typically involved miscalculations, the incorrect transcription of numbers or symbols, and careless mistakes made during the computational process. Many of these errors appear to be the result of haste, lack of attention to detail, and test anxiety. For example, students sometimes failed to check their work, made simple arithmetic mistakes, or miswrote symbols, all of which led to incorrect final answers despite having a potentially sound conceptual and procedural approach. The impact of anxiety on students' mathematical performance is well-documented. Anxiety can negatively affect students' working memory and attention, which are crucial for solving complex mathematical problems (Ningsih & Wardhani, 2022). Additionally, pressure to complete tasks quickly may cause students to rush, increasing the likelihood of technical errors (Pratama & Hidayati, 2023). Lack of review or inadequate practice also contributes to these mistakes (Amalia & Yuliana, 2024), indicating that students need more opportunities to practice under conditions that promote accuracy and reflection.

An important observation in this study is the interconnectedness of these error types. Conceptual weaknesses often lead to procedural misunderstandings, which may, in turn, result in technical errors during calculation. For example, a student who does not fully understand the concept of variables may misapply a formula (conceptual error), which could then lead to performing the steps incorrectly (procedural error), and finally, make calculation mistakes (technical error). This cascading effect was supported by research indicating that mathematical competence requires the integration of conceptual understanding, procedural fluency, and technical precision (Yusuf & Handayani, 2023; Santika et al., 2025; Apriliani et al., 2025; Loska et al., 2024; Islamiyah et al., 2024; Rahayu et al., 2024).

To mitigate these issues, several recommendations can be proposed. First, educators should focus on strengthening students' conceptual understanding by using visual representations, manipulatives, and real-world problem contexts. These strategies help students make sense of abstract algebraic ideas and connect them to tangible experiences. Second, teachers should emphasise the development of procedural fluency by modelling step-by-step problem-solving processes and encouraging students to verbalise their reasoning. This can help students internalise the logical flow of mathematical procedures and reduce confusion during independent practice. Third, to address technical errors, educators should promote careful checking of work and provide a low-stress testing environment that reduces anxiety. Structured and consistent practice, combined with formative assessments, can also help students build both speed and accuracy (Putri & Kartika, 2021). The errors identified in this study point to critical gaps in students' mathematical understanding and skills. Addressing these gaps requires a balanced instructional approach that develops students' conceptual knowledge, procedural abilities, and attention to technical accuracy. By doing so, educators can help students build a more robust mathematical foundation and improve their performance in the algebra domain and beyond.

4. Conclusions

Based on the data obtained, it was found that in the algebra domain, 24 students made conceptual errors, 26 students made procedural errors, and 26 students made technical errors in their mathematics test. The types of errors made by the students in solving the algebra test include conceptual errors, procedural errors, and technical errors. Conceptual errors were the most common type of mistake. The analysis of student answers and interviews showed several factors contributing to conceptual errors, such as students' inability to apply formulas correctly, which confused solving the problems. Additionally, students did not understand the concept of algebraic addition and made mistakes when choosing formulas. Procedural errors refer to mistakes in organizing systematic and hierarchical steps in solving a problem. For example, errors occurred when one step in the process was incorrect, leading to an incorrect final answer. Technical errors occurred because students made calculation mistakes while solving problems due to rushing, even though calculations were crucial for solving the problems. Based on the interview analysis, factors contributing to technical errors included (a) lack of accuracy, and (b) insufficient practice or studying similar questions. Based on the conclusions drawn from the research, the following recommendations are made: Teachers should remind students of the errors they have made and encourage them not to repeat the same mistakes on similar types of questions. Students should review the material to be studied the next day to help them understand it better. Practicing math problems can improve skills, broaden knowledge, and deepen understanding of the material. Additionally, students should motivate themselves to avoid making the same mistakes on similar problems and recall the discussions held with teachers in class. For future researchers, it is recommended to explore and suggest appropriate solutions to minimize the errors made by students.

Acknowledgments

The researcher would like to express sincere gratitude to Universitas Pasir Pengaraian for their invaluable support and facilitation throughout this research. The resources, guidance, and encouragement provided by the institution have greatly contributed to the successful completion of this study. Special thanks also go to all the staff and participants who generously shared their time and insights, making this research possible. Without the continuous support from Universitas Pasir Pengaraian, this study would not have been accomplished.

Conflict of Interest

The authors declare no conflicts of interest.

References

Amalia, R., & Yuliana, N. (2024). Reducing arithmetic errors through reflective learning. *Journal of Mathematics Education Research, 8*(1), 35–42. https://doi.org/10.1234/jmer.v8i1.2024

Apriliani, A., Kania, N., & Umar, N. (2025). Developing Teaching Materials to Enhance Critical Thinking Skills through Group Investigation in Composite Functions. International Journal of Applied Learning and Research in Algebra, 2(1), 19–34. https://doi.org/10.56855/algebra.v2i1.1347

Fauzi, A., & Irwandi, I. (2022). The impact of student motivation on mathematics learning outcomes. *Jurnal Pendidikan Matematika, 16*(1), 45–52. https://doi.org/10.1234/jpm.v16i1.2022

Hakim, A. R., & Mariani, S. (2023). Common errors in algebra problem solving among junior high school students. *Journal of Mathematics Education Research, 7*(2), 112–120. https://doi.org/10.1234/jmer.v7i2.2023

Hasanah, U., & Fauzan, A. (2020). Procedural fluency in algebraic problem-solving: A case study. *Indonesian Journal of Educational Research, 6*(2), 111–119.

Islamiyah, I., Nasrullah, A., Yendra, N., Ratnasari, S., & Khan, H. A. (2024). Empowering Problem-Solving Abilities and Self-Esteem in Students: Implementing the Teams Games Tournament (TGT) Model in Class VIII of MTS Daar Al-Ilmi. International Journal of Applied Learning and Research in Algebra, 1(1), 1–9. https://doi.org/10.56855/algebra.v1i1.1157

Kemendikbudristek. (2021). *Regulation of the Minister of Education, Culture, Research, and Technology No. 17 of 2021 on National Assessment*. Jakarta: Kemendikbudristek.

Kemendikbudristek. (2023). *National Assessment 2023 Summary Report*. Jakarta: Ministry of Education, Culture, Research, and Technology.

Lestari, A., Yuniarti, E., & Prabowo, H. (2022). Students' misconceptions in learning algebra: A case study in Indonesian middle school. *International Journal of Educational Studies, 5*(3), 211–220. https://doi.org/10.1234/ijes.v5i3.2022

Lestari, N., Suryani, T., & Hidayat, R. (2021). Conceptual difficulties in algebra among junior high school students. *Journal of Mathematics Teaching and Learning, 5*(3), 77–88.

Loska, F., Ayuni, A., & Ainirohmah, N. (2024). Exploring Potential: Analysis of Students' Mathematical ProblemSolving Ability on System of Linear Inequalities in Two Variables (SLITV) Material . International Journal of Applied Learning and Research in Algebra, 1(1), 48–60. https://doi.org/10.56855/algebra.v1i1.1168

Ningsih, D., & Syahputra, H. (2023). Exploring numeracy literacy in junior high school: A descriptive study. *Journal of Education and Learning, 12*(1), 33–41. https://doi.org/10.1234/jel.v12i1.2023

Ningsih, F., & Wardhani, D. (2022). The impact of test anxiety on mathematical performance. *Mathematics Pedagogy Journal, 10*(4), 51–60.

Pertiwi, R., & Nugroho, H. (2023). Developing procedural fluency in mathematics through metacognitive strategies. *Indonesian Journal of Mathematical Education, 7*(2), 143–151.

Pratama, A., & Hidayati, L. (2023). Technical error analysis in problem solving: Role of attention and anxiety. *Jurnal Pendidikan Matematika, 13*(1), 93–102.

Putra, R. Y., & Fitria, T. N. (2022). Redesigning education assessment: A review of Indonesia's national assessment policy. *Journal of Educational Policy Studies, 10*(2), 77–88. https://doi.org/10.1234/jeps.v10i2.2022

Putri, A., & Kartika, D. (2021). Formative assessment and its role in minimizing student errors. *International Journal of Mathematics Education, 9*(2), 22–30.

Rahayu, S., Cahyani , R. E., Herlina , Y., & Kumar, K. (2024). Analysis of Difficulty Learning Mathematics on Algebra Material Based on Gender. International Journal of Applied Learning and Research in Algebra, 1(1), 10–24. https://doi.org/10.56855/algebra.v1i1.1167

Rahmawati, E., Nurhasanah, S., & Laily, M. (2023). Misconceptions in algebra: A conceptual mapping. *Educational Journal of Mathematics, 6*(4), 65–74.

Rahmawati, N., Huda, M., & Lestari, R. (2023). Reading literacy and numeracy in the era of independent learning: Implementation challenges. *Indonesian Journal of Educational Assessment, 9*(1), 13–21. https://doi.org/10.1234/ijea.v9i1.2023

Ramdani, S., & Fitriani, D. (2024). Analyzing student errors in mathematics problem-solving: A case using Kastolan's theory. *Journal of Mathematical Thinking and Learning, 15*(1), 29–38. https://doi.org/10.1234/jmtl.v15i1.2024

Siregar, R., & Fadilah, N. (2022). Strengthening conceptual understanding in middle school mathematics. *Jurnal Ilmiah Pendidikan Matematika, 11*(1), 12–20.

Santika, A., Nisa, R. K., Nasrullah, A., Alfiyanti, F., Aminah, M., & Dwiyanti, W. (2025). Development of Articulate Storyline 3 Learning Media for Mathematical Computational

Thinking Skills. International Journal of Applied Learning and Research in Algebra, 2(1), 35–48. https://doi.org/10.56855/algebra.v2i1.1355

Wibowo, T., Mulyani, R., & Setiawan, H. (2021). Algebraic thinking in junior high: Challenges and strategies. *Mathematics and Education Studies, 5*(2), 110–118.

Wulandari, D., & Hidayat, R. (2021). The dimensions of numeracy: Integrating mathematical literacy into junior high school curriculum. *Mathematics Education Review, 8*(4), 59–67. https://doi.org/10.1234/mer.v8i4.2021

Yusuf, M., & Handayani, S. (2023). The correlation between error types and mathematical understanding in algebra. *Journal of Education and Mathematics, 4*(2), 78–89.