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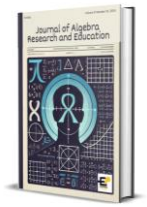

Students' Mathematical Reasoning Ability on The System of Linear Equations of Two Variables in Grade VIII

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Abstract

Mathematical reasoning ability is essential for students in learning mathematics. This study aims to describe students' mathematical reasoning ability in the topic of Two-Variable Linear Equation System (TVLES). The research employed a descriptive method with a qualitative approach. Data were collected through observation, tests, and interviews. The test instrument consisted of two validated essay questions focused on reasoning skills. The subjects were 20 eighth-grade students of SMP Negeri 4 Koto Gasib. Based on the test results, six students representing three ability levels (high, medium, and low) were selected for further analysis. The findings reveal that students with high initial ability were able to fulfill all indicators of mathematical reasoning, including expressing statements in writing, making conjectures, manipulating mathematical elements, constructing valid arguments, and drawing accurate conclusions. Students with medium ability were unable to express mathematical statements in writing and presented incomplete conjectures but demonstrated proper manipulation, provided some evidence, and drew generally correct conclusions. Meanwhile, students with low ability struggled to express written statements, showed incomplete reasoning in conjectures, and failed to construct valid arguments or conclusions, although they were able to manipulate correctly.

Keywords: Algebra; Learning Mathematics; Mathematical Reasoning Ability; Two-Variable Linear Equations.

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1. Introduction

Education plays a fundamental role in shaping individuals and society. According to Law No. 20 of 2003 on the National Education System in Indonesia, education is defined as "a conscious and deliberate effort to create a learning environment and learning process in which learners actively develop their potential to possess religious spiritual strength, self-control, personality, intelligence, noble character, and the skills needed by themselves and society" (Government of Indonesia, 2003). The Indonesian Dictionary defines "education" as a method, way, or act of guiding, derived from the root word *didik* (to educate) with the prefix *pe* and suffix *an* (Pristiwanti, Badariah, Hidayat, & Dewi, 2022). Teaching, in this context, is viewed as an intentional effort to influence behaviour and ethics, aimed at developing independence and maturity through instruction, mentoring, and coaching.

Among the various disciplines taught in schools, mathematics holds a critical position. It supports not only the development of science and technology but also serves as a foundational tool in everyday problem-solving and logical thinking (Siagian, 2016). Carl Friedrich Gauss referred to mathematics as "the queen and the servant of the sciences," underscoring its dual role as both an independent field and a supporting element for other disciplines (Wahyudi, Suyitno, & Waluya, 2018).

The importance of mastering mathematics is emphasised in Article 37 of Law No. 20 of 2003, which lists mathematics as a compulsory subject at both primary and secondary education levels (Afsari, Safitri, Harahap, & Munthe, 2021). To ensure effective learning, Bruner (as cited in Retnodari, Elbas, & Loviana, 2020) suggests that mathematics instruction should focus on understanding underlying concepts and their interrelationships. Mathematics learning is not just about memorising formulas or procedures; it involves building conceptual and procedural understanding that connects abstract ideas to real-world contexts, thus making learning more meaningful (Qamar & Riyadi, 2016).

According to the National Council of Teachers of Mathematics (NCTM), effective mathematics education should adhere to six guiding principles: equity, curriculum, teaching, learning, assessment, and technology. Furthermore, five key process standards are emphasized: problem-solving, reasoning, communication, connections, and representation (Putri, Sulianto, & Azizah, 2019). These standards highlight the critical role of mathematical reasoning a core cognitive ability that enables students to make logical inferences, justify solutions, and apply mathematical ideas to various problems.

Reasoning is fundamentally a mental activity involving processes such as recalling, imagining, memorizing, associating meanings, conceptualizing, and predicting (Ario, 2016). Strong reasoning skills help students approach real-life problems with structured and logical thinking. However, several studies and observations indicate that students often lack adequate reasoning skills. For instance, observations at SMP Negeri 4 Koto Gasib revealed that many eighth-grade students struggle with solving reasoning-based mathematics problems, reflecting a broader national concern.

The 2019 Programme for International Student Assessment (PISA) revealed that Indonesia ranked 72nd out of 78 countries in mathematical literacy, indicating serious

challenges in achieving national educational goals in mathematics (OECD, 2019). This aligns with national competency standards, which expect students to process, present, and reason through both concrete and abstract domains.

One particularly challenging topic in eighth-grade mathematics is the system of linear equations in two variables (TVLES). According to the 2013 Curriculum, this topic is essential for developing logical thinking. However, studies have found that students often face difficulties in interpreting and solving TVLES problems, especially in the context of word problems (Lineaus, Saputra, & Maulidia, 2016). These difficulties are often attributed to weak reasoning skills, poor understanding of variables, and frequent errors in constructing mathematical models (Qamar & Riyadi, 2016). Linola, Wahyuni, and Rahayu (2017) found that students at SMAN 6 Malang displayed varying levels of mathematical reasoning ability, with 4% of students categorised as low. These students often performed incomplete mathematical manipulations and provided insufficient justification for their answers. Similarly, Rismen, Rezi, and Lestari (2020) found that most eleventh-grade science students at SMAN 1 Koto Salak demonstrated weak reasoning and communication abilities, with higher achievement only in tasks involving direct calculations or the use of formulas.

Given the importance of mathematical reasoning in understanding and solving TVLES problems, and considering the low performance of Indonesian students in this area, the researcher is interested in conducting a study titled: "Students' Mathematical Reasoning Ability on the Topic of Systems of Linear Equations in Two Variables (TVLES) in Grade VIII at SMP Negeri 4 Koto Gasib."

2. Methods

This study employed a descriptive method using a qualitative approach. The data collected were not presented in numerical form but were instead explained in descriptive terms based on the responses and behaviours of the research subjects. The focus was on gaining a deep understanding of students' mathematical reasoning abilities rather than measuring them statistically.

The research took place on May 20, 2024, at SMP Negeri 4 Koto Gasib. The participants consisted of 20 eighth-grade students from Class VIII-1. Data collection was carried out through three main techniques: observation, written tests, and interviews. Prior to conducting interviews, students were given a mathematical reasoning test consisting of essay-type questions. Based on their performance, students with relatively similar capabilities were identified, and six of them were selected as representative subjects for in-depth analysis. These six students were categorized into high, medium, and low ability groups to explore differences in reasoning abilities within the topic of Systems of Linear Equations in Two Variables.

The qualitative data analysis followed a systematic process consisting of four main stages: data reduction, data processing, data presentation, and conclusion drawing. In the data reduction stage, the researcher simplified and organised raw data, mainly students' written responses, to focus on relevant aspects of reasoning ability. Data processing involved organising and interpreting the information to reveal patterns and insights. The data were then

presented in a structured manner to support clear understanding and analysis. Finally, conclusions were drawn by comparing and contrasting the data, allowing the researcher to interpret students' reasoning skills within the given mathematical context.

To develop the test questions, the researcher identified key indicators to assess students' difficulties and reasoning skills related to Systems of Linear Equations in Two Variables. These indicators were designed to evaluate various aspects of mathematical reasoning, including the ability to express mathematical ideas both verbally and in writing, propose logical conjectures, perform mathematical operations, justify the correctness of solutions, and draw appropriate conclusions from given statements. These indicators served as the foundation for analysing each student's reasoning process throughout the study.

3. Results and Discussion

3.1 Results

This research aimed to describe in detail the mathematical reasoning abilities of sixth-grade students when solving contextual problems involving Systems of Linear Equations in Two Variables. The study utilised a qualitative descriptive method with test and interview techniques to obtain comprehensive data on students' reasoning processes. Six students were purposively selected to represent three levels of mathematical ability: two students in the high ability category (T1 and T2), two in the medium ability category (S1 and S2), and two in the low ability category (R1 and R2). The problems presented to the students required them to interpret word problems, define variables, build algebraic models, solve equations using appropriate methods, and draw and justify their conclusions.

3.1.1 High-Ability Students (T1 & T2)

Students in the high ability category showed strong and consistent mathematical reasoning across both TVLES word problems. In the first question involving fruit prices, both T1 and T2 successfully identified all relevant known values and the information being asked. They clearly defined variables for the unknown prices and constructed accurate algebraic equations to represent the relationships given in the problem. They employed systematic methods namely elimination and substitution to solve for the variables and interpreted the results correctly. In the second question concerning the dimensions of a rectangular table, they again defined variables appropriately, constructed a correct mathematical model using the relationship between length and width, and used the perimeter equation to solve the system of equations. Both students verified their results by substituting the obtained values back into the original equations to ensure correctness. In the follow-up interviews, they were able to explain their steps in detail, justify their reasoning logically, and express confidence in the validity of their answers. These findings indicate that students at the high ability level demonstrated full mastery of mathematical reasoning indicators, including problem representation, conjecture formulation, mathematical manipulation, justification, and conclusion drawing.

3.1.2 Medium-Ability Students (S1 & S2)

Students in the medium ability group demonstrated partial success in solving the TVLES problems. Both students were able to understand the context of the problems and recognized

the need to form equations from the given information. However, they often neglected to write down the known quantities and what was being asked explicitly, which reflects a weakness in the representation aspect of mathematical reasoning. Although they showed some difficulty in accurately defining variables, both S1 and S2 managed to build appropriate mathematical models that aligned with the problem requirements. They applied solution techniques such as elimination and substitution, arriving at the correct final values. However, their steps were sometimes unclear or incomplete. For instance, S1 was able to verify the solution and gave an oral explanation during the interview, although the written justification was not fully developed. S2, on the other hand, was less able to articulate reasoning and did not verify the solution. Their overall reasoning processes indicated a fair level of mathematical manipulation skills, yet lacked structured justification and comprehensive conclusions. Thus, while medium-ability students demonstrated competence in procedural aspects of solving systems of equations, they struggled with clearly communicating their thought processes and justifying their results.

3.1.3 Low-Ability Students (R1 & R2)

Students categorised as having low mathematical ability exhibited significant challenges in completing the TVLES tasks. In both questions, they did not write down the known information or identify what was being asked, suggesting a major gap in problem representation. Furthermore, they did not explicitly define variables before attempting to solve the problems, which contributed to an unstructured and often confusing problem-solving process. Despite this, both R1 and R2 were surprisingly able to construct correct mathematical models by directly translating the word problems into equations, and they applied elimination and substitution techniques to obtain correct numerical results. However, their work lacked essential reasoning components. They neither verified their answers nor provided written or verbal conclusions. During interviews, they demonstrated uncertainty about the steps they used and were unable to justify their solutions, revealing a reliance on procedural trial and error rather than conceptual understanding. This suggests that while they possessed basic computational skills, their reasoning remained superficial and undeveloped, particularly in the areas of explanation, validation, and reflection.

In summary, the results show a clear stratification in mathematical reasoning abilities across ability levels. High-ability students were able to meet all indicators of reasoning with accuracy and clarity. Medium-ability students showed strength in mathematical manipulation but lacked structured justification and expressive clarity. Low-ability students could perform procedural tasks but were deficient in constructing, expressing, and validating their reasoning processes.

3.2 Discussion

The present findings highlight distinct variations in the quality of mathematical reasoning across different student ability levels, which align with recent scholarly work focusing on algebraic reasoning within TVLES (Two Variables Linear Equations System) contexts. Notably, high-ability students exhibited comprehensive competency across all evaluated reasoning indicators. These indicators include representation, conjecturing, procedural manipulation,

justification, and conclusion drawing. This observation is consistent with the research conducted by Rahma Tresna and Effendi (2023), who emphasized that students who effectively integrate conceptual understanding with procedural fluency tend to demonstrate superior performance in algebraic reasoning tasks. This suggests that a balanced mastery of both conceptual and procedural knowledge is crucial for high-level mathematical reasoning.

In contrast, medium ability students, while generally successful in solving problems correctly, showed considerable difficulties particularly in representing problem situations and verbally justifying their reasoning. This partial proficiency mirrors findings from Purnomo et al. (2022), who documented that students within the middle ability tier frequently overlook essential aspects of communication and justification in problem solving processes. Such gaps in expressing mathematical reasoning indicate a need for instructional strategies that specifically encourage verbal and written articulation of problem-solving steps to bridge these deficiencies.

Low ability students, although capable of applying procedural steps correctly, demonstrated significant shortcomings in deeper reasoning components, especially regarding presenting problem information, formulating conjectures, and drawing logical conclusions. This pattern aligns with the results of Meidiana et al. (2021), who found that students engaged in emergent modelling often possess operational understanding but struggle with clarity and completeness in reasoning expression. This reinforces the argument that procedural skills alone are insufficient and must be supplemented with activities that foster critical thinking and reasoning articulation.

Instructional approaches that emphasize contextual and meaningful learning environments, such as Problem Based Learning (PBL) integrated with ethnomathematical perspectives, have been shown to significantly enhance mathematical reasoning across various ability levels (Maidiyah et al., 2021). Such approaches situate mathematical problems within culturally relevant contexts, increasing student engagement and promoting deeper understanding. Furthermore, indicator-specific analyses reveal that justification and conclusion drawing are consistently the weakest reasoning components, especially among lower-achieving students. These findings underscore the urgent need for pedagogical interventions that explicitly focus on these critical reasoning skills rather than solely on procedural accuracy (Purnomo et al., 2022).

Self regulated learning is another pivotal factor influencing reasoning quality. According to Rahmatika and Waluya (2023), students exhibiting higher self-regulation tend to fulfill reasoning indicators more comprehensively, whereas those with lower self-regulation show overall weaker reasoning capabilities. This highlights the importance of cultivating metacognitive strategies and learner autonomy to improve mathematical reasoning outcomes.

Additionally, learning models such as Contextual Teaching and Learning (CTL) have been empirically demonstrated to bolster reasoning ability by embedding problems in real-world contexts and encouraging active student participation (Nurhusain et al., 2022). This context rich environment aids students in connecting abstract mathematical concepts to tangible experiences, thereby fostering more meaningful and robust reasoning.

Another key insight pertains to the critical role of representational skills. Research indicates that students who are adept at clearly expressing problem information and defining

variables are more likely to develop robust and effective solutions. This conclusion is supported by literature advocating the use of multiple representations in algebra learning, which enhances comprehension and problem-solving skills (Wikipedia, 2025; Sa'dah et al., 2025; Mulya et al., 2024; Cahayanik & Nurrahmawati, 2025; Putri, et al., 2024).

Innovative pedagogical tools, such as game-based learning applications, have also shown promise. For example, an Android-based TVLES game study revealed that students often performed better on tasks requiring conclusion drawing than on tasks involving conjecturing, suggesting the need for targeted support for weaker reasoning indicators (Purnomo et al., 2022). Finally, resource-based and open-ended teaching approaches, which encourage exploration, representation, and justification of reasoning, have been found to significantly improve reasoning outcomes across diverse learner populations (Susilawati et al., 2025).

In summary, the findings emphasise that high-ability students demonstrate strong, well-rounded mathematical reasoning, while medium-ability students often lack clarity in expression and justification despite procedural accuracy. Low-ability students rely heavily on procedural methods without adequate verification or conclusion. Consequently, instructional strategies must prioritize communication skills, representational competence, and scaffolded justification to enhance reasoning for all students, fostering a more equitable and effective mathematical learning environment.

4. Conclusions

Based on the results of the analysis and the results of interviews conducted by the researcher, it can be concluded that the mathematical reasoning ability of students in solving problems on the material of two-variable linear equation systems of students VIII 1 SMP Negeri 4 Koto Gasib is that students with high initial abilities are able to meet all indicators of mathematical reasoning ability. Students with moderate abilities are not yet able to present mathematical statements in writing, are able to submit conjectures but in their steps are not complete, are able to manipulate well, are able to compile evidence for the truth of the solution and are able to draw conclusions correctly and precisely. Students with low abilities are not able to present mathematical statements in writing, are able to submit conjectures but in their steps are not complete, are able to manipulate correctly, but are not yet able to compile evidence for the truth of the solution and finally students are not able to draw conclusions from the answers they have obtained.

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Conflict of Interest

The authors declare no conflicts of interest.

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