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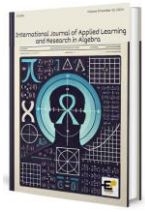

Linear Algebra Education in University: A Literature Review

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Abstract

The objective of this study is to conduct a thorough literature review in order to give a comprehensive understanding of the opportunities and challenges associated with the teaching and learning of linear algebra in university environments. The review assesses the pedagogical strategies used by academic institutions to enhance student engagement and comprehension, along with the application of linear algebra. We conducted a four-stage process to analyze five articles indexed in Scopus. The review emphasizes the importance of incorporating technology and real-world applications into linear algebra instruction to promote student motivation and active learning. It underscores the necessity of pedagogical approaches that promote student agency, particularly in online learning environments, where traditional teaching practices frequently persist. Exploring the correlation between learning outcomes and semantic networks, evaluating the impact of online platforms on student participation and agency, and evaluating real-world applications for teaching linear algebra are all potential future research directions.

Keywords: Education; Linear Algebra; University.

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1. Introduction

University students often struggle to master linear algebra due to its abstract nature and the complexity of its principles. Many students find linear algebra challenging, often due to their insufficient understanding of basic algebraic concepts. This makes them frustrated and less interested in learning (Wahyuniar, 2023; Rahim et al., 2020; Ferryansyah et al., 2018). The abstract concepts of matrices and eigenvalues impede understanding, as students find it challenging to comprehend and apply these ideas in practical applications (Thomas & Stewart, 2011). Furthermore, the pedagogical methods utilized in instructing linear algebra might profoundly influence student learning outcomes. Inquiry-based instruction and the utilization of visual tools such as GeoGebra have demonstrated efficacy in improving comprehension. However, not all students derive equal advantage from these approaches (Aytekin & Kıymaz, 2019; Haider & Andrews-Larson, 2022). The absence of appropriate teaching methodologies and the inadequacy of instructors might intensify students' challenges, resulting in a loop of subpar performance and diminished motivation (Rochana et al., 2020; Tristanti & Nusantara, 2023).

Linear algebra is a fundamental subject in higher education mathematics, essential for areas such as engineering, computer science, and economics. Their importance is in equipping students with critical abilities for problem-solving and modeling real-world scenarios, especially through concepts such as systems of linear equations and matrix operations (Ramírez-Montes et al., 2021; Darmayanti, 2023). Research demonstrates that a robust understanding of linear algebra is essential for students engaged in advanced mathematics and other disciplines, as it forms the foundation for numerous higher-level concepts and applications (Luneta & Legesse, 2023; Rensaa et al., 2019). Furthermore, the instructional methods employed in teaching linear algebra can profoundly affect students' comprehension and involvement. Research underscores the efficacy of incorporating inquiry-based learning and technology into the curriculum, which can improve students' conceptual comprehension and provide a more dynamic educational atmosphere (Cooley, 2016; Rahim et al., 2020). The cultivation of reflective abstraction abilities in students is crucial for the mastery of linear algebra, as these skills facilitate the connection between abstract concepts and practical applications (Kariadinata, 2020; Ferryansyah & Sari, 2021).

The aim of this study is to gain a general understanding of the challenges and opportunities associated with the application of linear algebra in universities through a literature review. We anticipate that academics and researchers will benefit from this in their linear algebra research context.

2. Methods

This study aims to review existing research to investigate how academic institutions employ linear algebra. This literature review includes previous university investigations related to linear algebra. We have selected five articles from Scopus for this review. Four stages are necessary to conduct a literature review on linear algebra utilization in universities. The initial stage is to identify articles that are pertinent to the application of linear algebra in universities (Rahmawati et al., 2022; Wahyuniar, 2023). The second phase involves the examination of pertinent concepts in linear algebra (Oliveira, 2012; Wang & Zhao, 2023). The third stage involves the synthesis of information regarding the application of linear algebra in universities (Chernov et

al., 2018; Açıkyıldız & Kösa, 2021). The fourth stage is to derive conclusions from the findings (Mkhize et al., 2014; Prokopenko et al., 2014).

3. Results and Discussion

Table 1 summarizes the research objectives, methods, and primary findings of each source. We assigned a code between A1 and A5 to each of the five examined files, selecting them based on the study's objectives. These codes will facilitate easier reference and comparison across the different studies. Each file's unique code will allow us to track specific methodologies and results as we synthesize the overall findings of the research.

Table 1. Summary of Linear Algebra education in university

Source	Purpose	Method	Findings
A1 (Wallach & Kontorovich, 2024)	To understand how the affordances of online communication platforms can shape interactions between students and instructors in online mathematics tutorials	The researchers: - translated and transcribed tutorial sessions, paying attention to mathematical narratives - used problem discussions as units of analysis - adapted Sfard's (2008) construct of Learning Teaching Agreement (LTA) to analyze discourse leadership, the duties offered to students, and the nature of the expected change	- Three types of instructional interactions were identified: lecture-ish, instructor-oriented, and cross-student - The lecture-ish format was the dominant type, suggesting that online communication platforms can influence instructional interactions and potentially limit opportunities for student-centered learning
A2 (López-Díaz & Peña, 2022)	To analyse the effectiveness of a seminar introducing linear algebra concepts in the context of a population migration problem	The researchers: - developed a seminar consisting of 10 sessions on applications of linear algebra in engineering, including the population migration problem - conducted anonymous questionnaires and personal interviews to assess student	- The population migration problem successfully integrated linear algebra concepts across four main topics: algebraic structures, vector spaces and linear applications, reduction of linear applications, and resolution of linear discrete dynamical systems - The problem was well-received by students, with positive

Source	Purpose	Method	Findings
		appreciation of the problem	feedback on the application of linear algebra concepts to a real-world context.
A3 (Christianson, et al., 2020)	To study the structure and topological development of semantic networks of mathematical concepts in linear algebra textbooks	The researchers: - constructed semantic networks from 10 linear algebra textbooks - used a modified RAKE algorithm to identify significant phrases representing mathematical concepts - connected concepts based on co-occurrence within sentences - employed persistent homology to track the growth and development of topological cavities in the networks	- Semantic networks in textbooks exhibit structural order, specifically core-periphery structure and community structure within the periphery - The core concepts, representing fundamental topics, are introduced earlier than peripheral, more specialized concepts - Fewer knowledge gaps, represented by topological cavities, exist in textbooks compared to null models, indicating a deliberate effort to connect concepts and facilitate learning - The persistence and number of knowledge gaps may be related to textbook difficulty and reader preferences
A4 (Bennett et al., 2022)	To describe the Data Analytics Research (DAR) course at Rensselaer Polytechnic Institute, a course-based undergraduate research experience (CURE) in applied data analytics, and its	The researchers implemented a project-based learning (PBL) approach in DAR, involving: - real-world, open-ended projects sourced from the Data INCITE Lab - interdisciplinary teams - a focus on data	- The DAR course effectively engaged students in data analytics research and application development, as evidenced by student projects, survey responses, and the impact on student

Source	Purpose	Method	Findings
	role in a data analytics pipeline for student success	visualization and communication - continuous formative and summative assessment using R notebooks, presentations, and peer feedback	career trajectories - The data analytics pipeline, including an introductory data analytics course and DAR, increased student interest and confidence in data analytics, attracting a diverse group of students to the field
A5 (Jung & Nardelli, 2020)	To introduce an information-theoretic framework for constructing optimal personalized explanations in explainable machine learning (ML)	The researchers: - developed a probabilistic model incorporating features, predictions, and user summaries to represent user background knowledge - defined optimal explanations as those maximizing the conditional mutual information (MI) between the explanation and the prediction, given the user summary - proposed an algorithm for computing optimal explanations based on i.i.d. samples of features, predictions, and user summaries	- The framework allows for constructing personalized explanations tailored to individual user knowledge, enhancing explainable ML by accounting for user background information - Algorithm 1 demonstrates the practical implementation of this approach - Numerical experiments using a computer vision task illustrate the framework's ability to provide user-specific explanations

This literature review looks at five different sources (A1.pdf to A5.pdf), each of which investigates a different aspect of learning, teaching, and research approaches across a variety of fields of study.

3.1. Linear Algebra, Data Analytics, and Online Learning are All on The Move

These two sources concentrate on instructional approaches and the impact they have on students' learning. The purpose of this source (A1) is to study how online communication platforms, notably Zoom, can influence the interactions between students and instructors in

linear algebra lessons. The researchers recognized three types of interaction patterns: lecture-like, instructor-oriented, and cross-student. The sessions primarily followed a lecture-like pattern, suggesting that online platforms may inadvertently restrict opportunities for active student interaction. Despite the intended student-centered structure, this remained the case. This article (A2) examines the Rensselaer Polytechnic Institute's Data Analytics Research (DAR) course.

This course is a course-based undergraduate research experience (CURE) curriculum that focuses on applied data analytics. The course takes a project-based learning (PBL) method, in which students collaborate with members from different fields to complete projects that are based on real-world scenarios. The purpose of this technique is to enable students to enhance their skills in data analytics, to foster creative problem-solving, and to inspire additional study about the topic. The researchers highlight the success of DAR in engaging students and building a data analytics pipeline that begins with an introductory data analytics course and continues in the research experience. This pipeline provides students with the opportunity to collect and analyze data.

3.2. Linear Algebra and Semantic Network Applications in the Real World

In addition, two further sources focus on the representation and application of mathematical principles. The source (A3) analyzes the efficiency of a seminar that introduced linear algebra concepts through a population migration problem. This seminar enhances the learning experience by incorporating real-world applications, targeting students in their first year of engineering schools. The researchers found that students positively received the population migration issue, which successfully incorporated fundamental linear algebra principles, leading to an increase in both their motivation and comprehension.

Source (A4) employs a network science methodology to investigate the structure and topological growth of semantic networks derived from ten linear algebra textbooks. The study revealed the presence of a core-periphery structure in these networks. The textbooks introduce the core, a collection of fundamental concepts, earlier than the peripheral, a collection of more specialized concepts. In addition, the researchers utilized persistent homology, a tool derived from algebraic topology, in order to discover knowledge gaps that were represented by topological cavities in the networks. The researchers found fewer gaps in the textbooks compared to the null models, indicating a deliberate pedagogical strategy to promote idea connectivity and enhance learning.

3.3. Explain Machine Learning (ML)

In its final form, Source (A5) offers an information-theoretic framework that explainable machine learning can use to produce the most effective personalized explanations. To represent an individual's background knowledge, the researchers constructed a probabilistic model that takes into account features, forecasts, and user summaries. The definition of optimal explanations is those that maximize the conditional mutual information (MI) between the explanation and the prediction, taking into account the user's level of comprehension throughout the process. The goal of this method is to personalize explanations by adapting them to the specific areas of expertise and requirements of each individual user. This study presents a theoretical framework and an algorithm, demonstrating the method's application through a computer vision problem.

Conclusions

The review of linear algebra education in universities and colleges underscores the necessity of pedagogical approaches that enhance student engagement through the use of technology and real-world applications. Instructors need to create learning environments that encourage student agency and active learning, as online platforms may reinforce traditional teaching practices. It is possible to enhance student motivation and comprehension by incorporating real-world applications into instruction. In the future, research should investigate the relationship between student learning outcomes and semantic networks, evaluate real-world applications for teaching linear algebra, and investigate the impact of online learning environments on student participation and agency in linear algebra courses. It is possible to create more engaging and effective learning experiences for linear algebra students by analyzing the interaction between pedagogy, technology, and knowledge representation.

Conflict of Interest

The authors declare no conflicts of interest.

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