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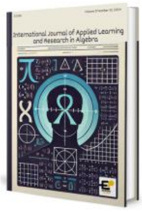

Students' Layer of Mathematical Understanding on the Material of Linear Equation One Variable in Terms of *Self-Directed Learning*

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Abstract

This study describes students' mathematical understanding when working on mathematical understanding test questions on the material of linear equations of one variable. The research method used in this research is descriptive. This research's data collection technique involves tests and interviews. The research subjects were selected from VIII E students of SMP Negeri 4 Tasikmalaya, as many as three people that fill one category each of high, medium, and low, and the results of the mathematical understanding. Data analysis techniques were used: data reduction, data presentation, and conclusion drawing. The results showed that subjects with a high self-directed learning category (S28) could work on problems with coherent, structured, complete answers up to the investing layer with intervention at the inventing layer. Subjects with moderate self-directed learning category (S31) can work on problems with correct answers up to the structuring layer with intervention at the noticing property layer. Subjects with a low self-directed learning category (S6) can work on the problem but not completely until the image-making layer with intervention on the image-making layer.

Keywords: Linear equation one variable; Mathematical understanding; Pirie-Kieren; Self-Directed Learning.

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1. Introduction

Mathematical understanding is an important mathematical ability students possess because it is an important foundation for solving mathematical problems or contextual issues. Mathematical understanding is crucial so that students not only memorize the material provided but also students better understand the concept of the learning material provided (Wulandari & Pujiastuti, 2020). In line with the opinion of Hidayat & Nuraeni (2022) Which states that mathematical understanding refers to students' knowledge of concepts, principles, and procedures and the ability to use solving strategies in solving mathematical problems. Mathematical understanding is important in mastering mathematics material because mathematics learning contains a lot of information, such as symbols or formulas, that will help students understand concepts algorithmically (Jusra & Liddini, 2022). Thus, mathematical understanding is vital in helping students solve mathematical problems through understood concepts.

Students' mathematical understanding is still relatively low. This is per the results of research by Nuraeni et al., (2018) Which states that mathematical understanding in MT students is low. This is in line with what was conveyed by Agustina & Qohar (2020), that in general, students' mathematical understanding is still low and characterized by errors in solving mathematical problems. Students' lack of mathematical understanding will cause them difficulty solving math problems. (Davita et al., 2020). In addition, this low mathematical understanding will cause low student learning outcomes, as revealed by Ilmiyah et al., (2021), that mathematical understanding is directly proportional to mastery of mathematics. Students' mastery of receiving subject matter can be seen from the grades obtained by students (Ayuwardani, 2023).

The material of linear equation one variable is an abstract mathematical concept and is generally only introduced in middle school as a basis for learning more complex concepts. In line with the opinion of Khairat et al., (2022) The material of one variable linear equation is the most difficult for grade seven junior high school. Supported by the opinion according to Fitriani (2018), which states that the material of a one-variable linear equation is problematic in mathematics because it usually involves story problems, where learning requires high ability. Sari et al., (2022) Several students have mentioned difficulties in solving math problems, namely in interpreting the problem's meaning, transforming sentences into mathematical models, and determining the formula used. Alaiya et al., (2024) Stated that students' low ability in problem-solving indicates that they have not been able to achieve each layer of Pirie-Kieren's mathematical understanding. Therefore, these indications need to be studied more deeply.

Mathematical understanding can be characterized as leveled but non-linear. It is a recursive phenomenon, and recursion occurs when thinking moves between levels of sophistication... Indeed, each level of understanding is contained within succeeding levels. Any particular level, dependent on the forms and processes within and further, is constrained by those without (Pirie & Kieren, 1988). Pirie and Kieren (1988) presented mathematical understanding into eight layers, including *primitive knowing, image making, image having,* *property noticing, formalizing, observing, structuring, and inventing*. The growth of mathematical understanding is not always in one direction, from the inner layer to the outer layer. When someone gets stuck in solving a problem, they may go back to their inner layers to gain a better understanding. In Piere-Kieren's theory, this is referred to as *folding back.* Another feature of Pirie-Kieren's theory is the intervention. Interventions are internal or external actions to stimulate a person's understanding (Mardiana et al., 2017).

Mathematical understanding is not only formed by the teacher's role, such as listening to explanations, memorizing mathematical formulas, and solving steps, but also by students' efforts to understand the meaning of the concepts learned. (Davita et al., 2020). One of the internal factors that can affect students' mathematical understanding, namely psychological aspects related to student learning independence (Regina et al., 2021) Alternatively, in this study, it is referred to as *self-directed learning. Self-directed learning* can be defined as the nature, attitude, and ability of students to carry out learning activities independently or with the help of others based on their motivation to master a specific competency so that it can be used to solve the problems they encounter (Rachmawati, 2010), cumulative learning habits will foster a strong desire to learn and form strict, resilient, responsible, and high-achieving individual (Winda et al., 2023). In line with the opinion of Li & Wu (2023), which states that students' *self-directed learning* can motivate them to learn, and the higher the tendency to learn independently, the better the learning outcomes. Thus, as mathematical understanding is directly proportional to the grades obtained by students, *self-directed learning* is essential for students to support their mathematical knowledge.

An earlier observation showed that students in grade VIII E SMP Negeri Tasikmalaya had a low understanding of mathematics. Some students have difficulty solving math problems, especially story-shaped problems involving linear equations of one variable. Data on student test scores support this statement, which is still below the standard criteria. The research explored the growth of mathematical understanding through self-directed learning on the Linear Equation One Variable (PLSV) material in class VIII SMPN 4 Tasikmalaya. It is expected to contribute to studying the growth of understanding in linear equation one variable.

2. Methods

The type of research carried out in describing the mathematics understanding of class VIII SMPN students regarding linear equation one variable material was a descriptive research method that used a qualitative descriptive approach. In line with the opinion according to Sudjana & Ibrahim (2012), revealed that descriptive research methods with a qualitative approach are used for researchers who want to describe or explain an event or phenomenon in sentences or words that are arranged and meaningful, not in the form of numbers. According to Bogdan and Taylor (Murdiyanto, 2020), qualitative methodology is a research approach that produces descriptive data in written or spoken words from people and observed behavior.

This research was carried out at SMPN 4 Tasikmalaya in July 2024, the odd semester of the 2024/2025 academic year. The research subjects were 3 class VIII students. The instruments used in this study use researchers as the main instrument because researchers function to select research subjects, collect data, analyze data, and conclude research results. Then, the supporting instruments used a self-directed learning questionnaire, one mathematical understanding test question on the material of linear equations of one variable in the form of descriptions to determine the layer of understanding achieved by students, and interviews. An expert validator has validated this research instrument, declared it valid, and can be used. The self-directed learning questionnaire used in this study is an adapted questionnaire from Williamson (2007). This questionnaire contains 55 indicators of awareness, learning strategies, learning activities, evaluation, and interpersonal skills.

The indicators and achievement of mathematical understanding indicators used by researchers are as follows:

Table 1- Indicators and achievement of mathematical understanding indicators

Questions	Layers of Understanding on Pirie Kieren's Theory	Achievement Indicator
A student cycles from home to school at 10 km/h. After a certain distance, his bicycle tire is flat, and he has to walk at 4 km/h. The total distance from home to school is 12 km, and the trip takes 1.5 hours. a. Sketch out the problem! b. What do cycling and walking travel the distance? c. If the student carries a tire repair kit and it takes 15 minutes to repair the tire, what cycling speed (for the rest of the trip) is required for the student to still reach school in 1.5 hours, including the time to repair the tire?	Primitive knowing (understanding the new definition, action of presenting the definition)	The subject can mention the information in the problem
	Image making (subject makes sense of prior knowledge and applies it to new knowledge)	The subject can generalize the distance traveled by bicycle in a variable and find the distance traveled on foot
	Image having (subject makes a picture related to the problem)	The subject searched for travel time by cycling and travel time by walking.
	Property noticing (subject combines aspects of the problem to form properties that are relevant to the problem)	The subject made a linear equation of one variable, namely metententen $x/10 + (12 - x)/4 = 1{,}5$
	Formalizing (subject makes abstractions of mathematical concepts based on properties that appear)	The subject solved the equation to produce the value of a variable representing the distance traveled by the bicycle.
	Observing (The subject coordinates formal activities at the formalization layer to be able to use them on related problems he faces)	The subject substituted the value of the variable representing the distance traveled by bicycle into the equation to produce the distance traveled on foot.
	Structuring (subject relates the relationship between one theorem and another and can prove it with logical arguments)	The subject solved the problem coherently, structured, and complete.
	Inventing (The subject has a structured understanding and creates new questions after solving the problem to answer the question "what if")	The subject can create a new equation from the problem in a different situation.

Adapted from (Susiswo et al., 2023).

The activity in this study is for students to fill out a self-learning questionnaire, which is carried out to determine the level of self-directed learning or student learning independence. Furthermore, the selected students were given mathematical understanding test questions on the Linear Equations of One Variable material. Researchers conducted unstructured interviews to learn more about the layers of mathematical understanding. Through a series of activities, qualitative data that was initially scattered and piled up can be simplified to make it easier to understand. Before conducting data analysis, it is important to ensure the data collected is valid. In this study, researchers used Miles and Huberman's data analysis technique through three stages: data reduction, data presentation, and conclusion drawing/verification.

3. Results and Discussion

3.1. Result

The results of this study describe students' mathematical understanding of the material of linear equations one variable in terms of high, medium, and low self-directed learning categories. The questionnaire was distributed three times to see the consistency of the results. After that, the results showed that seven students had consistent questionnaire results, while the other students' answers were inconsistent. The seven students consisted of two students in the low category (S6, S16), three students in the medium category (S17, S25, S31), and two students in the high category (S28, S30). Furthermore, these seven students were given mathematical understanding test questions. The results of the mathematical understanding test of prospective subjects are presented in the following table.

Table 2 - Results of the mathematical understanding test

Subject	Self-directed learning category	Indicator							
		PK	IM	IH	PN	F	O	S	I
S6	Low	✓	✓	✗	-	-	-	✗	-
S16	Low	✓	✓	✗	-	-	-	✗	-
S17	Medium	✓	✓	✓	✓	✗	-	✗	-
S25	Medium	✓	✓	✓	✓	✗	✗	✗	-
S28	High	✓	✓	✓	✓	✓	✓	✓	✓
S30	High	✓	✓	✓	✓	✓	✓	✓	✗
S31	Medium	✓	✓	✓	✓	✓	✓	✓	-

The selected subjects have consistent self-directed learning categories (high, medium, low categories), fulfill most indicators in the mathematical understanding test, and can express their opinions or thoughts verbally so that the subjects can provide maximum information. Based on the results of the mathematical comprehension test and interview, the selected research subjects are S6, which is a subject with a low self-directed learning category; S31, which is a subject with a medium self-directed learning category; and S28, which is a subject with a high self-directed learning category.

The Subject can express simple knowledge by writing what is known and what was asked, but it is incomplete. The subject did not write down how much travel time cycling and walking. Based on the interview results, the subject can explain well what he understands from the problem. Although he missed writing down one question, the interview confirmed that the subject could understand the meaning of the problem instructions that had to be solved. The subject also initially understands the material related to the problem, namely the relationship between speed, distance, and time. The subject can sketch the problem from the problem and complete it with a description. The subject can represent and explain what is thought and understood from the meaning of the problem by pouring it into a picture. The subject also began to understand the problem more deeply and build a picture of the solution by normalizing the cycling mileage in a variable and normalizing the other mileage from the understanding that the sum of the two distances is 12 km.

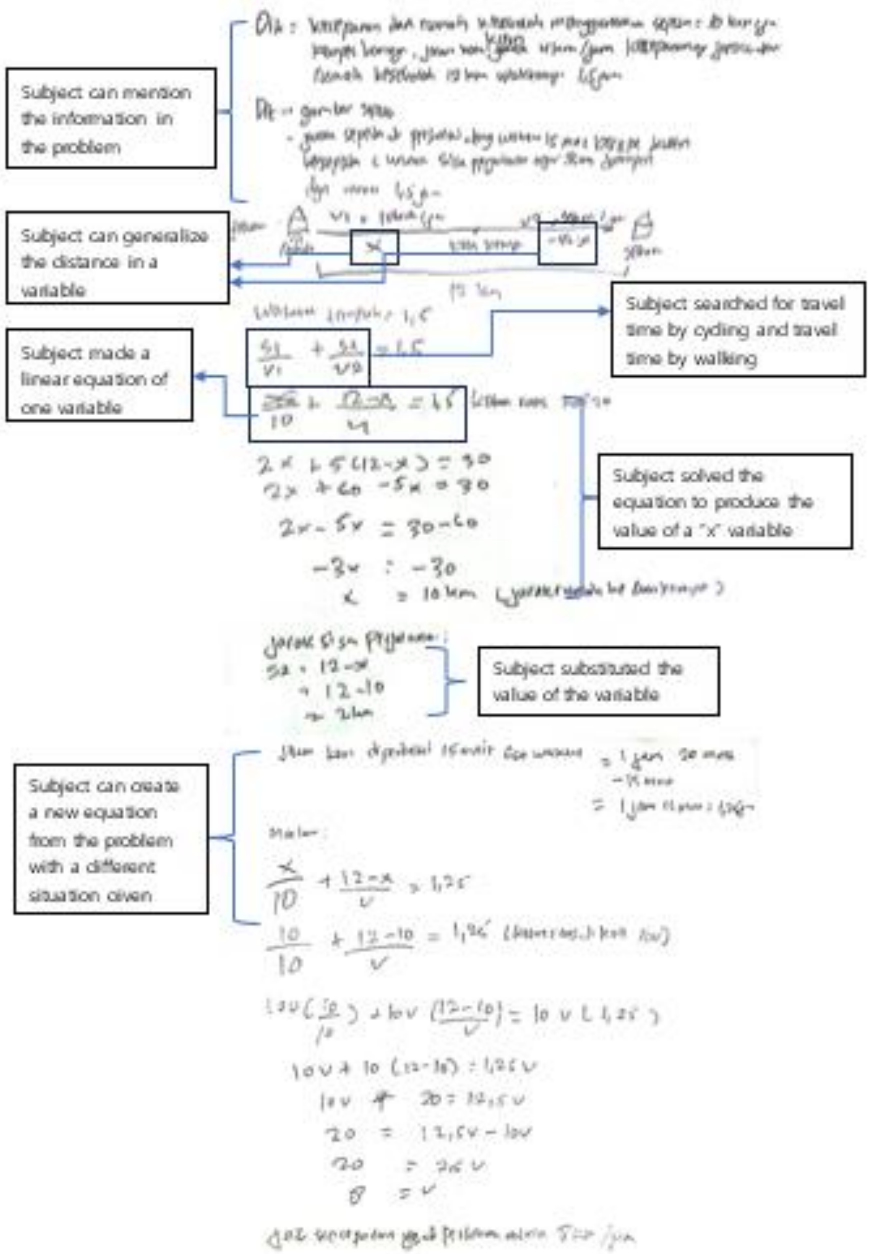

Figure 1 Answer result from S28

Based on Figure 1, the subject can understand previous knowledge, namely generalizing with variables and using it in new knowledge. Then, the subject constructed the one-variable linear equation by adding the cycling and walking travel times. This shows that the subject went through the process of finding the travel time of bicycling and walking. The subject explained that he could find the travel time for cycling and walking using the speed, distance, and time relationship formula he knew. However, the subject did not know the meaning behind the working steps.

The subject stated that he searched for travel time based on the information he had seen. The Subject could see the relationship of the known information to form a one-variable linear equation, one of the materials in the scope of algebra. The subject combined the bicycle travel time information with the walking travel time previously found to form a one-variable linear equation. The subject just realized that the cycling and walking travel times he looked for in the previous layer can be used to create an equation. Hence, the subject understands the *image having* layer and the *property noticing* layer. Then, the subject solved the equation based on his learning experience so that he managed to produce the x value he was looking for. Based on this, the subject can recognize and understand algebraic forms of terms, coefficients, constants, and variables. Then, the x value was substituted to get another value, namely the distance walked. The subject can explain the results of his work clearly and firmly. Then, in the third question, the subject thinks that solving the problem is still the same as the previous solution but feels confused about realizing it. However, after being helped with questions that stimulate his understanding, the subject can find the idea of solving it.

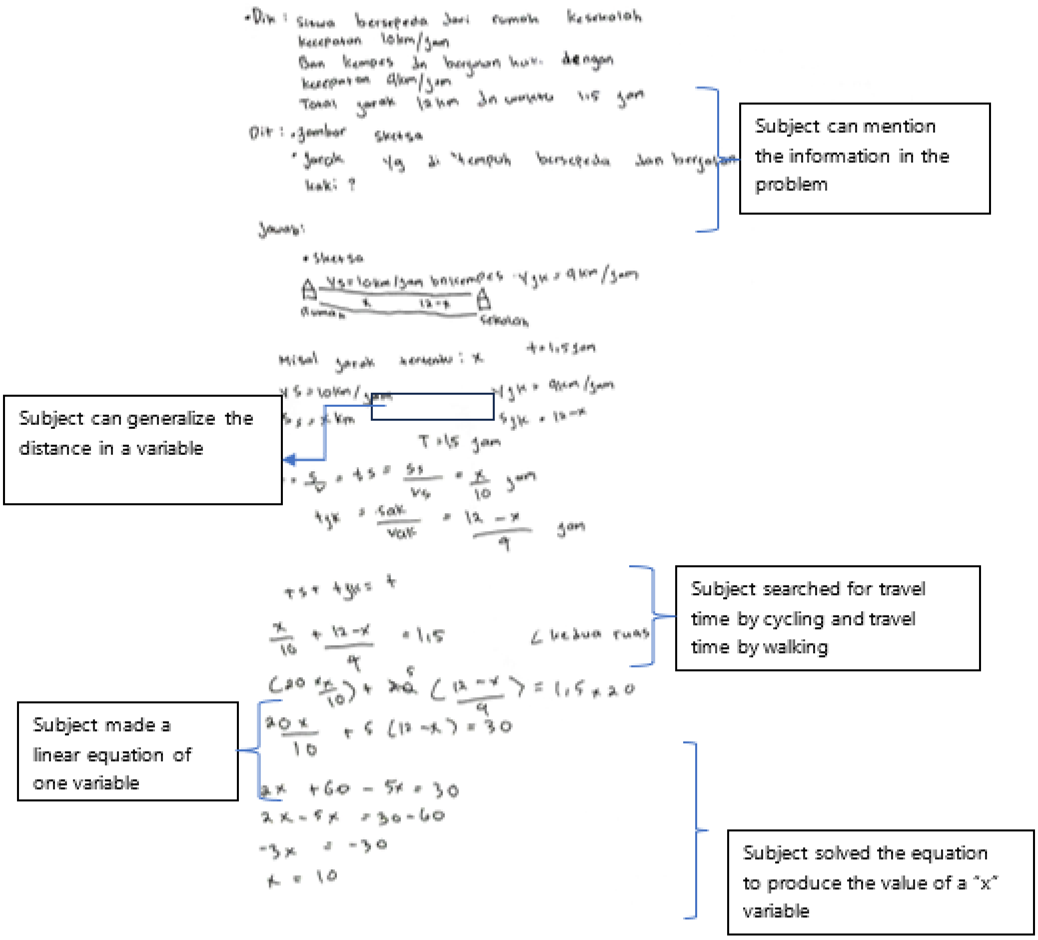

Figure 2 Answer result from S31

Based on Figure 2, S31 can write down the information in the problem. The subject also wrote what was asked in the problem but did not write down one more thing that was asked. Based on the interview, the subject can explain the known information well and entirely, even though, in his answer, he missed writing the question. This shows that the subject understands the meaning of the problem. The subject also has prior knowledge about the material related to the problem, namely the relationship between speed, distance, and time. Subjects can sketch the problem from the problem along with the information known in the problem. The subject can represent what is thought and understood from the meaning of the problem by pouring it into a picture. The subject also began to understand the problem more deeply and build a picture of the solution by normalizing the cycling mileage in a variable and normalizing the other mileage. However, initially, the subject's understanding of this layer was incomplete because the subject experienced folding back from the image-having layer to the image-making layer. The *folding back* process was done well so that the subject found the idea of the following solution step: finding the travel time of cycling and walking. The subject had no idea of the next step of the solution.

Therefore, the subject was given an intervention to move to the outer layer. Based on the intervention, the subject can form a one-variable linear equation by summing the bicycle and walking travel time information equal to the total travel time. Subjects can use the proper solution steps to produce the variable value. This shows that the subject can apply the properties of linear equations of one variable and the operation of similar and dissimilar terms appropriately to produce the correct answer. This shows that the subject can recognize and understand algebraic forms such as terms, coefficients, constants, and variables. The problem-solving step is not yet complete. There is still a distance to walk that needs to be found. Here, the subject was less careful in paying attention to the question asked, so he did not write the answer. However, after being interviewed directly, the subject can answer and find the distance walked. This shows that the subject can solve the problem completely. The subject can relate what is known with concepts that can help him solve the problem. S31 can use the concept of the relationship between speed, distance, and time to find travel time, then form a one-variable linear equation through intervention, solve the equation, and substitute the value obtained to get another value to answer the question in the problem.

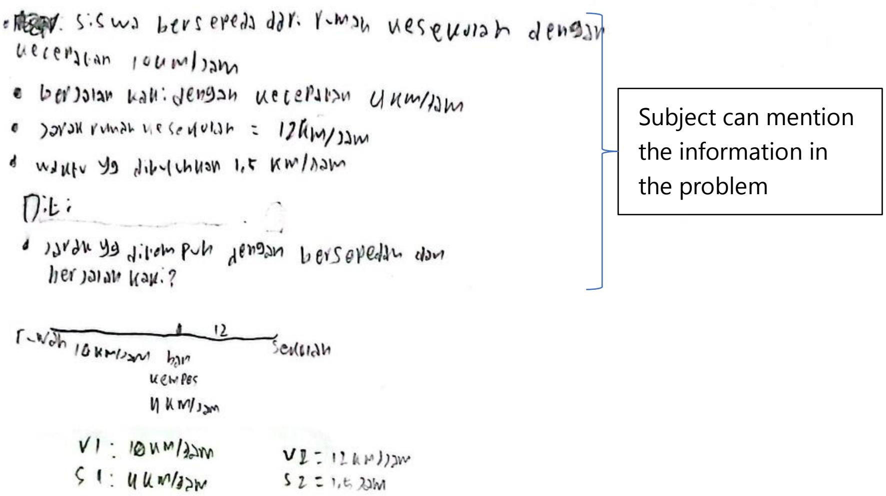

Figure 3 Answer result from S6

Based on Figure 3, S6 can write what is known and asked in the problem, such as cycling speed, walking speed, distance from home to school, and time needed. The subject made a mistake in writing the units of distance and time. Based on the interview, the subject could explain the known information well and realized his mistake in writing the units of distance and time. Although hesitant, the subject also explained that he remembered the material related to the problem, namely the relationship between speed, distance, and time. The subject can sketch the problem from the problem with the information known in the problem. The subject could represent what was understood from the meaning of the problem in the form of a picture. However, the subject wrote incorrect speed (v_2) and distance (s_1) information. This is because the subject is not careful about understanding the problem, but he realizes This mistake at this point, the subject seems to understand the problem more deeply. He could formalize the distance with variables. Therefore. Interventions are given to stimulate his understanding. The subject would not understand the generalizing idea in the first intervention, prompting questions. Therefore, the researcher told him to measure the subject to understand the steps. Furthermore, the intervention was given to find the distance walked by connecting and inking in new information with examples.

The subject was able to find the distance walked after the intervention. The interview also shows that the subject made mistakes in understanding the concept and meaning of the problem. The subject thought the value of x t capitalized was 6 m, and the rest was six; however, the subject realized his mistake after being given intervention. The subject understood to work on simple equations such as $2two + x = four$. e. The subject can recognize and understand algebraic forms such as terms, coefficients, constants, and variables. However, the subject may lack the ability to interpret the problem in an abstract form, such as generalizing with variables. Then, the subject could not find the formula for the travel time for cycling and walking. The subject has been given an intervention, but unfortunately, the subject cannot make a picture of the problem-solving.

3.2. Discussion

Based on the data explained above, the discussion regarding the analysis of mathematical understanding of the material of linear equations one variable in terms of high, medium, and low self-directed learning categories is as follows.

3.2.1. Analysis of S28 Mathematical Understanding with High Self-Directed Learning

Subjects with high *self-directed learning* (S28) at the *primitive knowing* (PK) layer can write down the known and questionable information in the problem and have prior knowledge about the relationship between speed, distance, and time. This shows that S28 has an initial understanding of the topic. In the *image-making (IM) layer, the subject can sketch the problem; this* shows that the subject represents what is thought and understood from the meaning of the problem. Actions that describe thoughts in mental images and image representations indicate the subject's understanding at the *image-making* level (Pratama, 2017). At this layer, the subject begins to build an image to understand the problem more deeply by normalizing the distance traveled by bicycle in a variable and finding an example of walking distance. At the *image having* (IH) layer, the subject can create an image related to the problem.

The subject knows the formula that can be used and has an idea to use the formula, which is to find the travel time of bicycles and walking. At the *property noticing* (PN) layer, the subject can create a one-variable linear equation based on the information sought in the previous solution. In this layer, the subject can combine the bicycle and walking travel times found through the *folding back* process to the previous layer to form a one-variable linear equation. In the *property noticing* layer, the subject tries to connect and combine the information obtained from the previous level (Pratama, 2017). This *folding back* process illustrates that students have the initiative and effort to solve problems. Wahyuni et al. (2020) revealed that students with high *self-directed learning* will have high responsibility, initiative, discipline, and creativity. At the *formalizing* (F) layer, the subject can solve the one-variable linear equation by applying the properties of the equation to produce the value of a variable representing the distance cycled. In this layer, the subject can also write more mathematical notations and symbols, use formal mathematical operations, and arrange them more precisely and structured.

The subject reached the *formalizing* layer because of the new knowledge obtained based on his attention to the rules, properties, and methods used to solve the problem (Pratama, 2017). At the *observing* (O) layer, the subject substituted the value of the variable representing the bicycle mileage into the equation to produce the walking mileage. This shows that the subject observes relationships or patterns in various mathematical concepts that have been learned. In this case, the subject saw that the concept of substitution could solve the problem. The subject can also explain it with logical arguments. At the *structuring* (S) layer, the subject can compile the steps of solving the problem in a structured and complete manner by involving the concepts of speed, distance, time, linear equations of one variable, and substitution. This shows that the subject can link the relationship between one theorem and another and can prove it with logical arguments. In the *inventing* indicator (I), through external intervention, the subject can create new equations from problems with different situations. This shows that the subject already has a structured understanding, and his understanding is not limited so that he can answer the question "*what if*." Thanks to his persistence, the subject can solve problems in the outermost layer. This is in line with the opinion of (2024), which states that students who have independence in learning are not easily discouraged or persistent in facing problems.

The subject worked on the problem well and tried hard to solve the problem. S28 did not give up, tried to understand the problem, and recalled the teacher's lessons. This shows that he has high motivation and a sense of responsibility for the tasks given. In his interview, he admitted that he was used to learning alone; therefore, he did not depend on others because he believed in his abilities. Supported by the opinion of Gusnita et al., (2021) which states that students with a high level of learning independence will try to complete the tasks given by the teacher with their abilities. In addition, during the interview, it was also seen that he could communicate well and express his opinion confidently.

3.2.2. Analysis of S31 Mathematical Understanding with Medium Self-Directed Learning

Subjects with moderate self-directed learning (S31) at the primitive knowing (PK) layer can write down the known and questionable information in the problem and have prior knowledge of speed, distance, and time relationships. At the image-making (IM) layer, the subject can represent what is known and asked in the problem in an image sketch. It can formalize the distance traveled by bicycle in a variable and find the equation of walking distance after going through the folding back process from the image having layer to image making. This is supported by the opinion (2005), which states that the outer understanding layer can support and provide information to deepen more basic activities or understanding. At the image having (IH) layer, the subject can make an image related to the problem; the subject can find the idea of finding the travel time of cycling and walking. Through external intervention, the subject can create a one-variable linear equation at the property noticing (PN) layer.

The subject can create a one-variable linear equation by combining the travel time of bicycling and walking equal to the total travel time. At the formalizing (F) layer, the subject can solve the equation to produce the value of a variable representing the cycling distance by applying the properties of one-variable linear equations and operating similar and dissimilar terms well. At the observing layer (O), the subject did not write down the problem's artistry. However, through the interview, the subject could answer questions related to substituting the value of the variable representing the distance traveled by cycling into another equation to produce the value of the distance traveled by walking. The subject was not careful in paying attention to the problem's question. In the structuring (S) layer, the subject could relate the relationship between one theorem and another and prove it with logical arguments. The subject could compile a structured and complete problem-solving, which included connecting the concepts of speed, distance, and time with one variable linear equation.

S31 did the problem well and tried to solve the problem even though there were questions that could not be answered. Based on his interview, S31 was quite confident when answering questions. Although he felt confused and still tried to work on the problem, his motivation to solve the problem was not too strong. When working on the problem, S31 also folded back several times, but it was less effective, so external intervention was needed. Although it takes time to reach a specific layer, this intervention is effective so students can reach the next layer. This is related to students' self-directed learning level in the moderate category, which shows that S31 still needs help in his learning activities. This is in line with the research results of Amaliyah et al., (2019) which states that students with moderate self-directed learning, when finding difficulties or not understanding the material, do not always try to find solutions from other learning sources but sometimes try to ask classmates who are considered to understand the material better.

3.2.3. Analysis of S6 Mathematical Understanding with Low Self-Directed Learning

Subjects with low self-directed learning (S6) at the primitive knowing (PK) layer can mention what is known and asked in the problem and write it on the answer sheet even though there is an error in writing the units of distance and time, but the subject can realize his mistake. In the image-making (IM) indicator, the subject can represent what he understands from the meaning of the problem in a picture. The submission of what the subject thinks is the beginning of development to the image-making layer (Pratama, 2017). Through external intervention, the subject can gain a deeper understanding, namely modeling bicycle mileage in a variable and finding a model for walking mileage. In the image having (IH) layer, the subject could not create an image related to the problem. The subject knew the related formula but did not know how to use it. Through external intervention, the subject knows the speed formula but cannot manipulate it to find the other formulas, namely distance and time, so the subject also cannot use the formula to find the travel time of bicycle and walking. Therefore, the subject could not move to the outer layer.

S6 could work on the problem but did not complete it and did not try to solve it. S6 admitted that she was confused and did not know how to solve the problem. S6 had difficulties solving the problem but did not try to find a solution. This shows that he lacks motivation, does not have confidence in his ability to solve problems, is not responsible for the tasks assigned, and tends to depend on the help of others, in line with the results of Kleden (2013), which states that students who lack the initiative to learn will have a high dependence on the teacher.

4. Conclusions

Students with high self-directed learning (S28) in solving linear equation one variable problem can reach the outermost layer, namely inventing. The subject folds back independently at the property noticing layer. The subject was given intervention at the inventing layer. Students with moderate self-directed learning (S31) in solving linear equation one variable problem can reach the structuring layer. The subject performs effective folding back at the image-having layer and is given intervention at the property-noticing layer. For students who have low self-directed learning (S6) in solving linear equation one variable problems, the subject can reach the image-making layer.

Conflict of Interest

The authors declare no conflicts of interest.

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